

Time-Reversal UWB positioning beacon for railway application

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Abstract—This paper studies a new localization system for railway transport using Ultra Wideband (UWB) radio and Time Reversal (TR) techniques. UWB radio has the potential to offer good performance in terms of localization precision. Time Reversal channel pre-filtering facilitates signal detection and also helps increasing the received energy in a targeted area. In this paper, we study the characteristics of time reversal temporal and spatial focusing. We analyze the contribution of time reversal associated with UWB in terms of localization error. To perform this study, a deterministic channel model, consisting of several reflected paths combined with the direct path is modeled. In terms of localization error, the simulation results show that TR-UWB technique delivers improved performance over the UWB alone localization approach.

Keywords-component; Ultra Wide Band (UWB); Time Reversal (TR); geometrical optical model; TDOA.

I. INTRODUCTION

Guided urban automated transportation systems are progressing significantly nowadays, highlighting many benefits. User's security and accessibility to these guided transport systems constitute a major issue dealt with many teams. In order to obtain an efficient and safe control and command system of the trains, it is essential to ensure an adequate exchange of information between vehicles and infrastructure and to determine accurately and constantly the absolute localization of the train.

The track to train data exchange is usually called CBTC for Communication Based Train Control. It can use many different radio techniques [1]. The localization system is based on proprioceptive sensors (phonic wheels, Doppler radars...) embarked in the train. This on-board system is coupled to the use of beacons located at ground, on the track, between the rails. These beacons are kilometre markers. They are used to compensate for the drift of the localization information computed using the proprioceptive sensors alone, when the train moves. The beacons provide absolute localization information whenever the train passes over them. They are adequately spaced along the track in order to limit the localization drift to a tolerable safe value. In many railway systems, these beacons constitute the only equipment remaining on the track [2]. Therefore, it could be interesting to

remove this last equipment from the track, for example to facilitate track maintenance.

To evaluate the feasibility of this removal, we examine a new solution based on the Ultra Wideband Impulse-Radio technology (UWB-IR) associated to the Time Reversal (TR) technique thus, constituting a new beacon realization. In this case, the beacon would be situated on the side of the track, a few meters away therefore, not interfering with track maintenance operations.

In this context, the proposed association between UWB and TR techniques for the precise localization of trains is illustrated in Fig. 1. It shows the particular case of a railway tunnel. The beacons are geo-referenced. TR-UWB beacons are installed on the side of the track. The TR-UWB beacons are kilometre markers. They can also exchange local CBTC information over the UWB radio link. When arriving in the range of the UWB communication, the train computes its absolute localization to the beacons using time of flight information. Localization algorithms such as TDOA (Time Difference Of Arrival) and TOA (Time Of Arrival) are considered. Moreover, multiple UWB transmitters, in a multiple input configuration, can be located in a beacon enhancing focusing and availability. The local Channel State Information (CSI) between any beacon transmitter and virtual optimal beacon localization along the track is identified a single time during the initial installation. This information is introduced as pre-filtering data in the different UWB transmitters. Therefore, focusing is obtained in the required position, along the track, whenever the train passes, improving the absolute localization process [3].

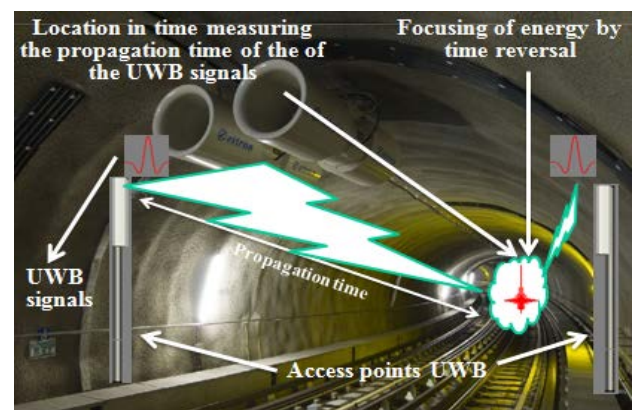


Figure 1. TR-UWB proposed localization system.

In a first part, this paper evaluates the temporal and spatial focusing performance of the time reversal (TR) method in a Single Input Single Output (SISO) configuration. In the second part, we analyse the contribution of TR on the quality of localization in one dimension. This contribution also aims to verify whether the combination of UWB and TR techniques allows obtaining the decimetre required localization level of precision in the railway application.

This paper is organized as follows: The second part presents the technique of time reversal and its main characteristics. The third part presents the propagation channel model based on a deterministic approach in a SISO configuration. The fourth section gives the evaluation results of the TR-UWB solution in terms of the temporal and spatial focusing. The fifth section presents the performance of the proposed solution in terms of localization error. Finally, section VI summarizes the results and presents future work.

II. CHARACTERISTICS OF TIME REVERSAL

Classically, Time Reversal has been applied to acoustics and underwater systems [4, 5], but recently, it has been studied for broadband and UWB communications. The first time-reversal experiment was reported for electromagnetic waves in the 2.45 GHz band [6]. This contribution suggests that the techniques developed for ultrasound might be used for the electromagnetic case. It is an interesting challenge because in many real environments (buildings or cities), microwaves using wavelengths between 10 and 30 cm are scattered off objects such as walls, desks, cars and so on, which produces a multitude of paths from the transmitter to the receiver. In such situations, a time reversal antenna should be able not only to compensate for the multipath effect, but also to increase the information transfer rate, thanks to the many reflections/reverberations that occur.

We propose to use the properties of time reversal for localization systems. As explained in [6], the general idea of Time Reversal is to compensate the effects of the propagation channel by inserting a pre-filtering stage at the transmitter level.

The principle of the proposed TR-UWB system uses three steps. Firstly, we select the signal we want to transmit. In our case, it is the second derivative of the Gaussian function; it is an ultra short pulse represented in Fig. 2. Secondly, the channel impulse response is measured between the transmitter (Tx) and the receiver (Rx) and the channel state information is loaded into Tx. Thirdly, the selected signal and the impulse response are reversed in time and transmitted by Tx in the propagation channel, up to Rx. This TR-UWB principle, represented in Fig. 3, can be mathematically described by noting $s(t)$ the transmitted pulse, $h(t)$ the complex impulse response of the channel and $h^*(-t)$ the complex conjugate of the time reversed version of $h(t)$. We note $y(t)$ the received signal without TR and $y_r(t)$, the received signal with TR at the receiver. Their expressions are given by:

$$y(t) = s(t) \otimes h(t) + n(t) \quad (1)$$

$$y_r(t) = s(t) \otimes h^*(-t) \otimes h(t) + n(t) \quad (2)$$

Where \otimes represents the convolution operation and $n(t)$ is the Gaussian noise.

From equation (2), we deduce the equivalent impulse response $heq(t)$ which corresponds to the autocorrelation function of the channel [7]:

$$heq(t) = h^*(-t) \otimes h(t) \quad (3)$$

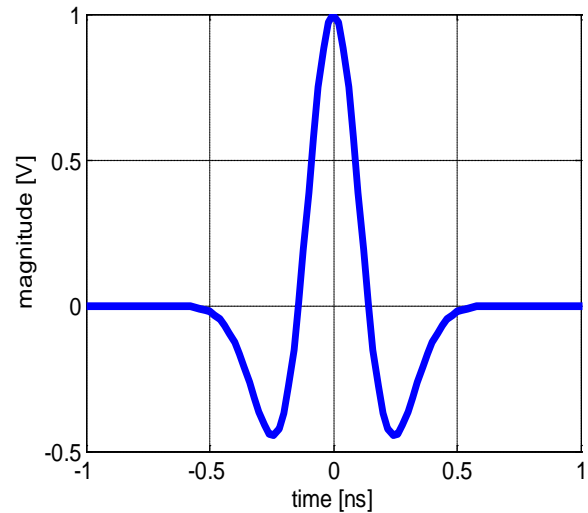


Figure 2. Transmitted pulse (second derivative of Gaussian function)

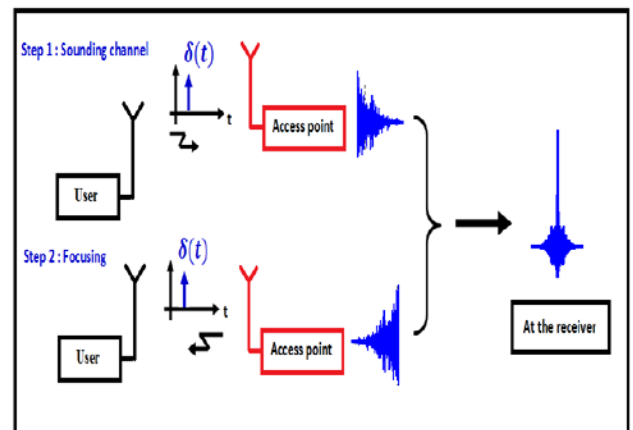


Figure 3. Principle of Time Reversal technique

The autocorrelation function is used to evaluate two main characteristics associated to time reversal, i.e., the temporal focusing and the spatial focusing. These characteristics are very beneficial to the UWB system [8]. To study the temporal focusing, we evaluate the Focusing Gain (FG), which is defined as the ratio of the strongest peak in TR received to the

strongest peak received by a conventional UWB system [9]. It can be written as:

$$FG_{[dB]} = 20 \log_{10} \left(\frac{\max(|y_{\pi}(t)|)}{\max(|y(t)|)} \right) \quad (4)$$

Higher FG could potentially translate into higher communication range and higher precision of localization for a localization system as compared to a pulsed UWB system.

The study of the Spatial Focusing (SF) is performed considering a TR-UWB system in a SISO configuration. The channel impulse response of the intended receiver which locates in position p_0 is noted $h(p_0, t)$. The channel impulse response of the unintended receiver located in position $p_i, (i \neq 0)$ is noted $h(p_i, (i \neq 0), t)$. The equivalent channel impulse response of the intended receiver is then given by:

$$heq(p_0, t) = h^*(p_0, -t) \otimes h(p_0, t) \quad (5)$$

While the equivalent impulse response of the unintended receiver is given by:

$$heq(p_1, t) = h^*(p_0, -t) \otimes h(p_1, t) \quad (6)$$

The spatial focusing is then evaluated by the ratio of the strongest peak power received by the intended receiver to the strongest peak received by the unintended receiver [10]. Mathematically, it can be written as:

$$SF_{[dB]} = 20 \log_{10} \left(\frac{\max(|heq(p_0, t)|)}{\max(|heq(p_1, t)|)} \right) \quad (7)$$

III. PROPAGATION CHANNEL MODELLING

The objective of this modelling is to characterize the channel impulse response. As stated before, TR would benefit from the complexity of the propagation environment. If the environment is increasingly complex, best should be the focus of energy [9]. Therefore, we propose to select a channel model that allows testing this TR feature by using progressively an increasing number of paths of the channel: 2, 3, 4, 6 and 10 paths.

To model the channel, we assume a directional antenna and we use a deterministic approach. We consider a two parallel infinite and flat dielectric walls corresponding to the floor and the ceiling shown in Fig. 4. T_x and R_x are separated by d_0 . Except for the direct path (LOS), the signals transmitted from T_x to R_x undergo reflections on the floor and ceiling.

These reflections can be divided into two categories; one is that the first reflection happens on the floor surface, the other one is that the first reflection happens on the ceiling. It should be noted that these two cases have common rules. Firstly, our interest will focus on case 1, where the first reflection happens on the floor. It has been shown that this model, although established based on case 1, is valid for case 2 as well [8].

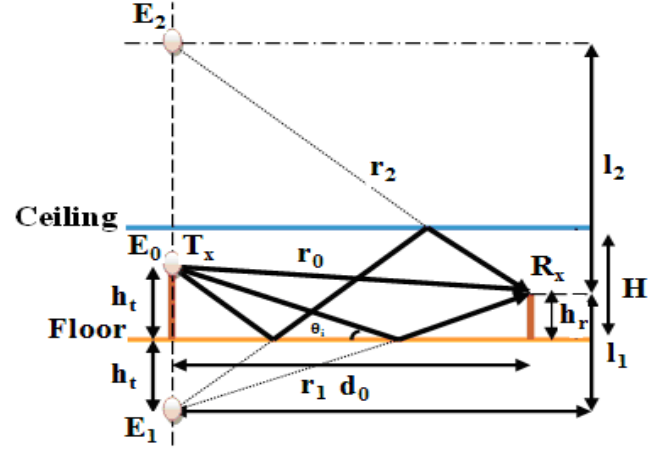


Figure 4. Channel model configuration (image method).

In Fig. 4, the number of paths corresponds to the number of reflections between the floor and the ceiling. The first path is the direct path r_0 , the second path is the path reflected on the floor; its length corresponds to r_1 . The third path is the path undergoing two reflections, one on the floor and one on the ceiling...

From this model it is possible to obtain the length of all the paths travelled by the transmitted wave, using the image method and the geometrical optical approach [11].

- Path N° 0: $r_0 = \sqrt{d_0^2 + l_0^2}$; with $l_0 = |h_t - h_r|$ (8)
- Path N° 1: $r_1 = \sqrt{d_0^2 + l_1^2}$; with $l_1 = |h_t + h_r|$ (9)
- Path N° 2: $r_2 = \sqrt{d_0^2 + l_2^2}$; with $l_2 = 2H + |h_t - h_r|$ (10)
- Path N° n: $r_n = \sqrt{d_0^2 + l_n^2}$; with $l_n = \begin{cases} nH + |h_t - h_r|^* \\ (n-1)H + |h_t + h_r|^{**} \end{cases}$ (11)

n corresponds to the path number or the number of reflections.

*: if n is even

** : if n is odd

In this step, we do not yet consider adding white Gaussian noise ($n(t) = 0$). Thus, the received signal is given by:

$$y(t) = y_d(t) + y_r(t) \quad (12)$$

$$y_d(t) = \left(\frac{\lambda}{4\pi r_0} \right) s(t - \tau_0) \quad (13)$$

$$y_r(t) = \frac{\lambda}{4\pi} \left(\sum_{n=1}^N [R_v(\theta_n)]^n \frac{s(t - \tau_n)}{r_n} \right) + \left(\sum_{n=1}^N [R_h(\theta_n)]^n \frac{s(t - \tau_n)}{r_n} \right) \quad (14)$$

Where, $y_d(t)$ corresponds to the received signal with the direct path. r_0 and τ_0 are respectively the distance travelled and the corresponding time of flight. $y_r(t)$ is the received signal constructed with the N reflected paths. λ is the wavelength. $R_v(\theta_n)$ and $R_h(\theta_n)$ are the reflection coefficients for vertical and horizontal polarizations; they are given by [12]:

$$R_v(\theta_n) = \frac{\cos \theta_n - \sqrt{\frac{1}{\epsilon_r} - \frac{1}{\epsilon_r^2} \sin^2 \theta_n}}{\cos \theta_n + \sqrt{\frac{1}{\epsilon_r} - \frac{1}{\epsilon_r^2} \sin^2 \theta_n}} \quad (15)$$

$$R_h(\theta_n) = \frac{\cos \theta_n - \sqrt{\epsilon_r - \sin^2 \theta_n}}{\cos \theta_n + \sqrt{\epsilon_r - \sin^2 \theta_n}} \quad (16)$$

ϵ_r is the relative permittivity of the floor or ceiling and θ_n is the incident angle.

For different rays, reflections angle θ is different. It is given by:

$$\theta_n = \arctan^{-1}\left(\frac{d_0}{\sqrt{l_n^2 + d_0^2}}\right) \quad (17)$$

r_n and l_n are respectively the distance travelled and the vertical distance between the image S_n and the receiver (Fig.4).

IV. TEMPORAL AND SPATIAL FOCUSING

For the simulations, we use the input parameters presented in Table I. For a SISO configuration composed of LOS (Line Of Sight) and 10 reflected paths, the impulse response without TR is shown in Fig. 5. Fig. 6 shows the equivalent impulse response after TR. Comparing this last equivalent impulse response to the impulse response without TR, we obtain a significant focusing gain. This focusing gain is calculated for SISO configurations with 2, 3, 4, 6 and 10 paths using equation (4). The results are presented in Table 2. In this case, these results confirm that TR performance takes advantage of the complexity of the channel.

The gain obtained using the two paths SISO configuration is 2.97 dB, increases to 5.92 dB for 4 paths and to 6.30 dB for 10 paths.

TABLE I. INPUT PARAMETERS FOR SIMULATION

Input parameter	
Waveform: $s(t)$	Second derivative of Gaussian (Fig. 2)
Reference Distance $T_x-R_x : d_0$	10 m
Height of Tx: h_t	3 m

Height of Rx : h_r	2 m
Height of ceiling : H	4 m

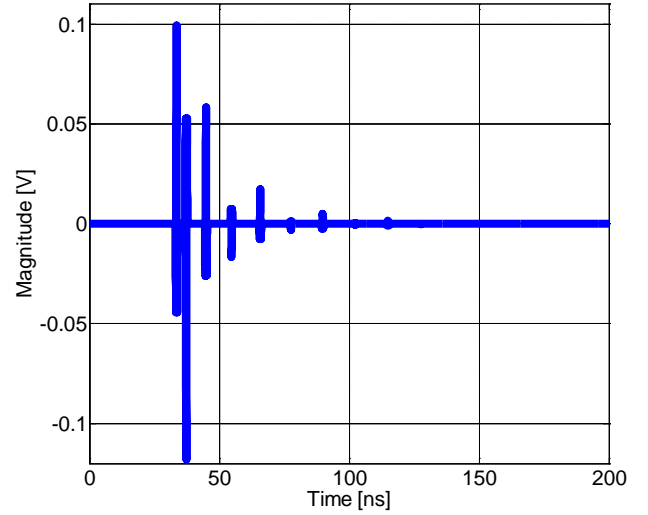


Figure 5. Channel impulse response without TR (SISO 10 paths)

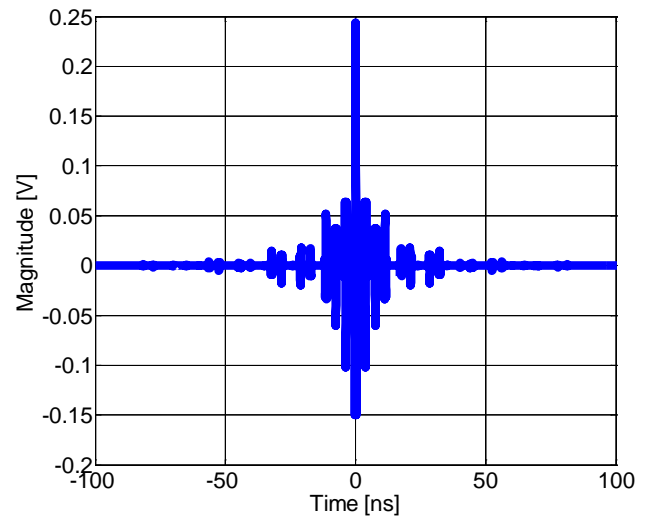


Figure 6. Channel impulse response with TR (SISO 10 paths)

TABLE II. FOCUSING GAIN PERFORMANCE EVALUATION

SISO Configuration	2 paths	3 paths	4 paths	6 paths	10 paths
FG [dB]	2.97	5.40	5.92	6.27	6.30

In Fig. 7, the focusing gain versus number of paths reaches a horizontal asymptote after 8-10 paths because the attenuation of the reflected signals becomes more and more important with the number of reflected paths.

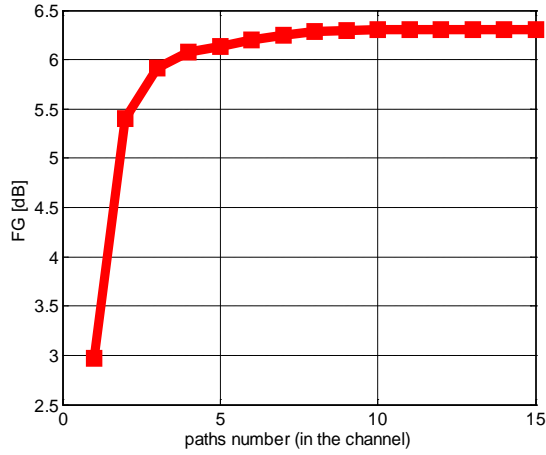


Figure 7. Focusing gain vs. path number

This FG property is also visible in Fig. 8 and 9, representing the PDP (Power Delay Profile) for the 2 and then 10 paths SISO configurations. The PDP for TR-UWB is given by:

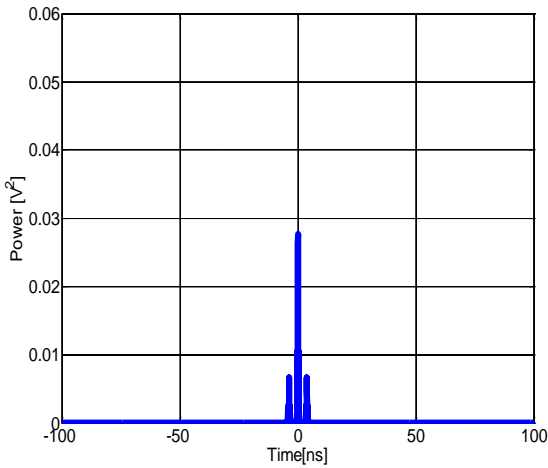


Figure 8. PDP for TR-UWB (SISO 2 paths)

$$PDP_{TR-UWB} = 10 \log_{10} |heq(t)|^2 \quad (18)$$

To study the spatial focusing SF, we first consider the SISO 4 paths configuration to calculate the equivalent impulse response ($heq(p_0, t)$) of the intended receiver corresponding to the reference distance $d_0 = 10 \text{ m}$ between T_x and R_x (Fig. 10). The equivalent impulse response ($heq(p_1, t)$) for the unintended receiver positioned in p_1 , at $d_1 = 20 \text{ m}$ is illustrated in Fig. 11. Moving from p_0 to p_1 , we obtain a loss of focus corresponding to 10.14 dB.

Then, a comparative study of the FS is done for the previous SISO 2, 3, 4, 6 and 10 path configurations. We move the intended receiver R_x from the target position $d_0 = 10 \text{ m}$ using a step of 0.1 m . The results are presented in Fig. 12. Moving the intended receiver on both sides of the reference position, we obtain a loss of focus of energy. In

addition, the FS curve becomes sharper and sharper as the number of paths increases.

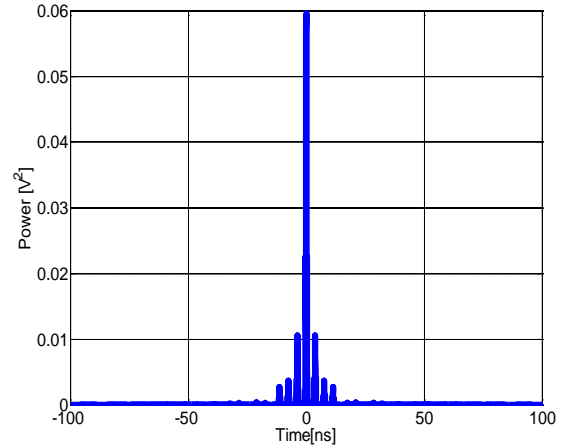


Figure 9. PDP for TR-UWB (SISO 10 paths)

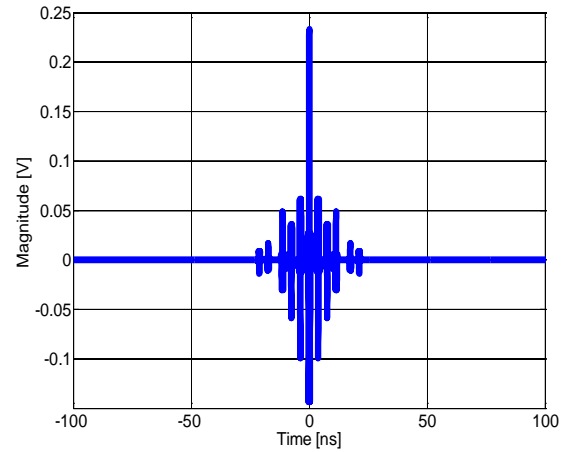
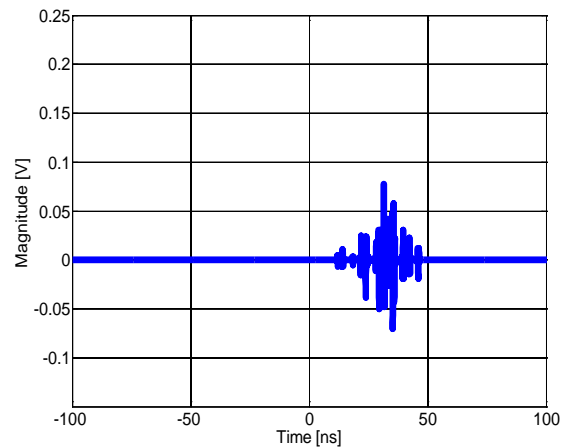


Figure 10. PDP for TR-UWB (SISO 10 paths)


 Figure 11. The unintended receiver $heq(p_1, t)$ ($d_1=20 \text{ m}$)

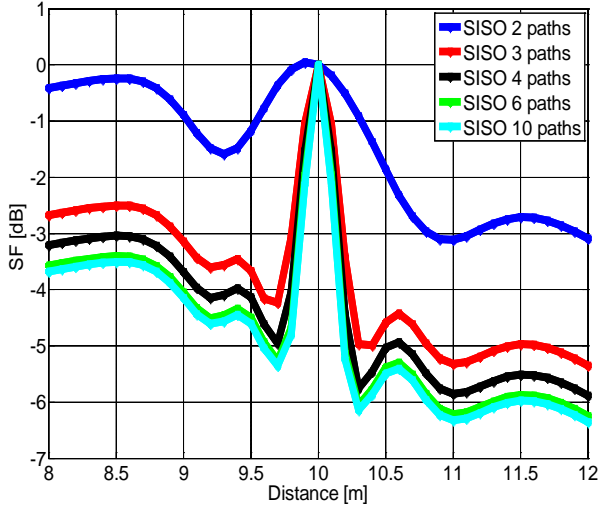


Figure 12. SF vs. the Tx-Rx distance for SISO 2, 3, 4, 6, 10 paths

V. TR-UWB-IR FOR LOCALIZATION

In this section, we evaluate the localization error considering two different cases: UWB Impulse Radio (UWB-IR) technology alone and UWB combined with time reversal (TR-UWB-IR). The goal is to estimate the contribution of TR to the location in terms of precision as a function of the propagation environment complexity. Using three base stations, we determine the position of a mobile in a 2D plane. To locate the mobile, each base station sends its own signal (recorded and reversed in time in the case of TR). The UWB signals constituted by a signal of second derivative of the Gaussian are modulated by an antipodal modulation and coded by a Gold code [13], one code per base station. Processing is performed at the mobile (R_i) to determine its position relative to the base stations. The distance error is given by the difference between the calculated position and the actual position of the mobile.

The mobile receives the signals from each base station and performs an adequate signal processing to determine its position, relative to the base stations. Using the TDOA technique, the signal received at the mobile is processed to retrieve the position of the latter [14]. The localization technique (TDOA) is combined with the Chan algorithm to calculate the position of the mobile [15]. This technique is illustrated in Fig. 13, where S_1 , S_2 and S_3 represent the three base stations of known coordinates, M is the mobile to be located. The difference distance between mobile and the i^{th} base station is given by:

$$R_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2} \quad (19)$$

Where, (x, y) are the unknown coordinates of mobile position, (X_i, Y_i) are the coordinates of the base stations.

Considering as a reference station S_1 (the reference station is the nearest station of the mobile, in the studied case), the difference distance between reference station S_1 and other stations is given by:

$$R_{i,1} = c \cdot d_{i,1} = R_i - R_1 = \sqrt{(X_i - x)^2 + (Y_i - y)^2} - \sqrt{(X_1 - x)^2 + (Y_1 - y)^2} \quad (20)$$

Where, c is the celerity of light, $d_{i,1}$ is the TDOA estimate between reference station and the i^{th} station. Calculating the differences in distance allows us to define a system of nonlinear equation of hyperbolas (equation 21), resolvable by the method of Chan.

$$R_{i,1}^2 + 2R_{i,1}R_1 = X_i^2 + Y_i^2 - 2X_{i,1}x - 2Y_{i,1}y + K_i - K_1 \quad (21)$$

For a three base station system, Chan's method produces two TDOA to determine the coordinates (x, y) of mobile:

$$\begin{bmatrix} x \\ y \end{bmatrix} = - \begin{bmatrix} X_{2,1} & Y_{2,1} \\ X_{3,1} & Y_{3,1} \end{bmatrix}^{-1} \cdot \left\{ \begin{bmatrix} R_{2,1} \\ R_{3,1} \end{bmatrix} R_1 + \frac{1}{2} \begin{bmatrix} R_{2,1}^2 - K_2 + K_1 \\ R_{3,1}^2 - K_3 + K_1 \end{bmatrix} \right\} \quad (22)$$

Where:

$$K_1 = X_1^2 + Y_1^2;$$

$$K_2 = X_2^2 + Y_2^2;$$

$$K_3 = X_3^2 + Y_3^2;$$

$$R_{2,1} = cd_{2,1};$$

$$R_{3,1} = cd_{3,1}.$$

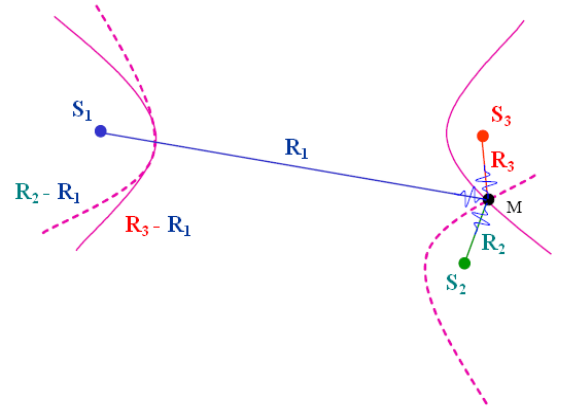


Figure 13. 2-D TDOA localization process

For the comparative study between the conventional localization system UWB-IR and the proposed system TR-UWB, we first use the case UWB-IR to locate the mobile then obtained information on the position of the mobile are used as reference for the purposes of locating with the TR-UWB system. Our comparison is based on the computation of the

Root Mean Square Error of localization between the conventional UWB-IR system and the proposed TR-UWB-IR system. We consider the SISO 10 paths channel configuration. An additive white Gaussian noise is also injected. Fig. 14 shows the comparative study of the RMSE for both systems. A signal to noise ratio (SNR) of 3 dB is considered in each case. In these conditions, we obtain a better precision of localization using the TR-UWB-IR system.

Trying to confirm these results, we then repeat this operation with our SISO 2, 6 and 10 path channels. A large number of iterations are used (1,000).

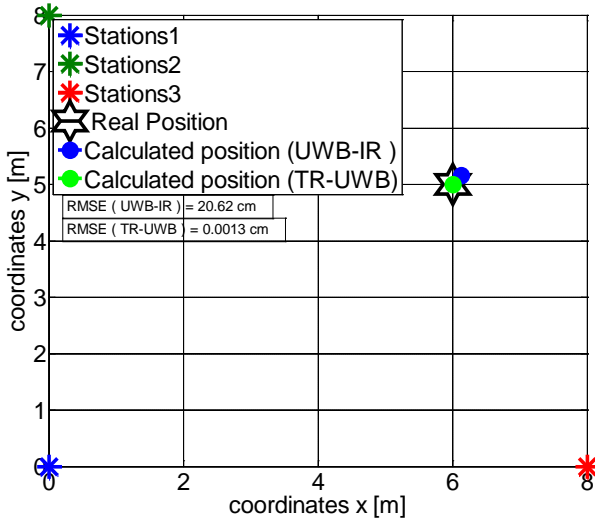


Figure 14. RMSE evaluation for conventional UWB-IR and TR-UWB (2 D positioning system)

Table III presents the simulation results in these three cases. They show that the proposed solution TR-UWB-IR gives repetitively the best performances in terms of localization. Indeed, in the studied case, the localization error is 20.62 cm in the case of SISO 10 paths without TR while it is only $130 \cdot 10^{-5}$ with TR.

These preliminary results show that the combination of UWB and TR techniques allows obtaining a more accurate localization that could be in line with the decimetre necessary level of precision required by the railway beacon application.

TABLE III. COMPARATIVE STUDY BETWEEN THE UWB IMPULSE RADIO (UWB-IR) AND UWB-TR IN TERMS OF FOCUSING GAIN AND ERROR LOCALIZATION (SISO 2, 6 AND 10 PATHS, SNR=3 dB)

SISO Config.	FG [dB]	RMSE UWB-IR [cm]	RMSE UWB with TR [cm]
2 paths	2.97	18.71	$\sim 45 \cdot 10^{-5}$
6 paths	6.27	19.40	$\sim 81 \cdot 10^{-5}$
10 paths	6.30	20.62	$\sim 130 \cdot 10^{-5}$

We then evaluate the Root Mean Square Error of localization (RMSE) versus SNR. The results are presented in Fig. 15.

The conventional UWB and TR-UWB results obtained for the 2, 6 and 10 paths configurations are very different with a significant advantage to the TR-UWB configurations. We note that for $\text{SNR} > 2 \text{ dB}$, the localization error remains fairly low. The extremely high performance of the proposed system on simulated data can be probably explained by the simulation scenario. Simulations using more complex scenarios are provided and experimentations in the anechoic chamber are underway to evaluate the results in the practical case

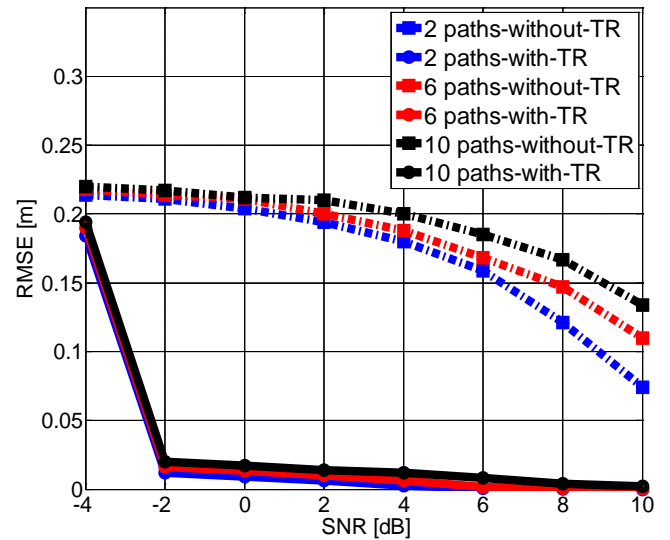


Figure 15. Comparison UWB-IR vs. UWB-IR/TR, RMSE vs. SNR for SISO 2, 6 and 10 paths

VI. CONCLUSION

In this contribution, we have presented results obtained related to the association of the UWB-IR and the time reversal technologies. The aim is to develop a new railway track localization beacon. In this railway scenario, the local Channel State Information (CSI) between any beacon transmitter and a virtual optimal beacon localization along the track is identified a single time during the initial installation. This information is introduced as pre-filtering data in the UWB transmitter. Then, focusing is obtained in a pre-defined zone in the beacon radio coverage area, along the track. Whenever the train repetitively passes in the beacon area, it computes its localization to the beacon. This information is improved by the use of TR. To evaluate the effectiveness of this association, we have selected a deterministic channel model that allows evaluating the potential improvement by using progressively an increasing number of communication paths of the propagation channel. The temporal focusing and the spatial focusing were calculated. A 2 D localization scenario has been modelled and evaluated. The results show that time reversal allows to

significantly decreasing the localization errors, especially in complex propagation channels. In future work, we will evaluate the performances of the proposed solution in the IEEE 802.15.3a and 4a channel model and validate these results through laboratory experimentation.

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