Wireless LAN based Indoor Positioning using Radio-Signal Strength Distribution Modeling

Yaemi Teramoto
Hitachi, Ltd.,
Central Research Laboratory
Kokubunji-shi, Tokyo, Japan
Email: yaemi.teramoto.xy@hitachi.com

Akinori Asahara
Hitachi, Ltd.,
Central Research Laboratory
Kokubunji-shi, Tokyo, Japan
Email: akinori.asahara.bq@hitachi.com

***Abstract*—Aiming to solve one of the serious problems of radio-signal strength (RSS) indoor positioning (namely, fluctuation of RSS values due to reflective waves), a novel indoor-positioning method based on RSS distribution modeling (called ”RSS distribution modeling using the mirror-image method,” RDMMI), was developed and evaluated. With RDMMI, a model of RSS distribution is created by using measured RSS and considering reflective waves. RDMMI achieves an average positioning error of 3.1 m, which exceeds that of conventional nearest-neighbor (NN) method by 0.6 m. It also accomplishes average positioning error of 3.7 m with a small set of randomly chosen RSS measurements. Furthermore, even in the case that radio source positions are unknown, RDMMI exceeds NN method.**

I. Introduction

Indoor-positioning systems are prevailing for providing location based services using location information of moving objects such as persons or devices.[1], [2] Location based services are needed both indoors and outdoors. The most popular location based service is indoor navigation.[3], [4] There are also needs for location based service to support operations of large-scale facilities like plants or logistics centers. For example, navigation while maintenance and inspection for industrial plants, optimizing an operation in logistics centers based on movement of workers and baggages, and evacuation guidance in commercial facility or public facility.

One of the important issues for indoor-positioning systems is constructing low-cost and commonly used infrastructure. Although, currently, there are no standards but various kinds of devices and methods for indoor-positioning, wireless-LAN based positioning system is likely to become a standard. Because, wireless-LAN devices are widely used for telecommunication and comparatively inexpensive.

In this article, a novel indoor-positioning method using widely-used Wi-Fi access points is proposed. Figure 1 shows a positioning system which the proposed method is based on. In this system, positioning devices (Wi-Fi access points) are installed in the positioning environment. A positioning object person has a receiver device such as a smartphone which is able to receive radio signal strength (RSS) from the access points. The receiver device can send received RSSs to a server.

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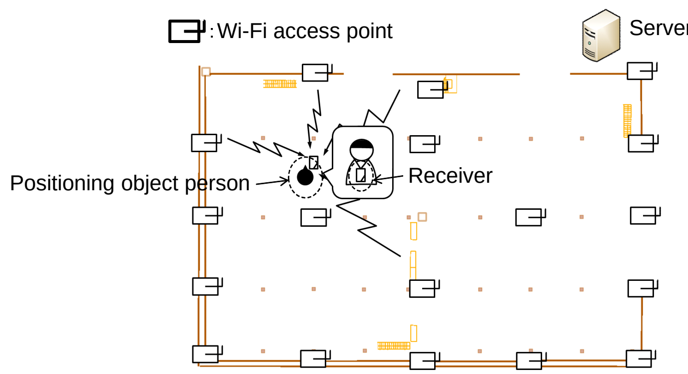

Figure 1. Positioning system

Regarding wireless RSS based indoor-positioning, there are two main approaches: trilateration based [5], and sampling based [5], [6], [7], [8], [9], [10]. By the trilateration based approach, the distance between a radio source and a receiver is calculated using a monotonically attenuating property of RSS according to the distance. Then a position where a RSS is measured is determined by means of a geometric approach. By the sampling based approach, a model which indicates a relationship between position and RSS is created by training model parameters using sampling data measured in a positioning field. There are mainly two kinds of modeling approaches: probabilistic modeling and propagation modeling. The former approach creates a probabilistic model which indicates probabilities of positions when a RSS is measured. Then a target position where a RSS is measured is estimated by the model. The simplest probabilistic-modeling method is the nearest-neighbor (NN) method. The NN method searches sampled RSS measurements for a position that has the nearest RSS values to presently measured RSS values. The latter approach creates a propagation model which describes how RSS attenuate with increasing distance from a radio source. In this approach, model parameters are also trained using sampling data.

The trilateration based approach requires positions of devices which are installed in an environment but does not require a set of RSS measurements which is previously measured at multiple points in a positioning area. A problem with trilateration based approach is that positioning accuracy decreases because of fluctuation of RSS values caused by radio wave reflection by obstacles such as walls, pillars, and human bodies.

The sampling based approach requires a set of RSS measurements but often doesn't need device positions. A problem with sampling based approach is that to achieve high positioning accuracy, many RSS calibration samples are needed to evenly measured in the positioning field.

Accordingly, in the present study, a new positioning method, using RSS-distribution modeling, was developed. With this method, RSS distribution is modeled by using RSS measurements while the radio-wave reflection effect is considered. The RSS distribution can be modeled with a small number of RSS measurements, even if the sample points are non-uniformly selected. Furthermore, positions of access points can be estimated as model parameters using the RSS measurements.

The rest of this paper is organized as follows. Several related works and our contributions are explained in Section 2. The RSS-distribution modeling method is described in Section 3. Evaluation results of the proposed method compared with the NN and trilateration methods are presented in Section 4. Finally, summary and future works are mentioned in Section 5.

II. Related works

A. Trilateration based approach

In the case of trilateration, RSS is monotonically attenuated according to the distance between a radio source (i.e. an access point) and a receiver. RSS can therefore be assumed to be a function of the receiver position, (x, y, z), and an RSS vector with the same number of dimensions as the number of receivers is given by

$$\begin{aligned}\mathbf{R} &= (R_1, R_2, \cdots, R_i) \\ &= (f_1(x,y,z), f_2(x,y,z), \cdots, f_i(x,y,z)) \end{aligned} \quad (1)$$

Each function, $f_i(x, y, z)$, is a monotonically attenuating function in which each access point's position is the highest RSS value. Here, if the distance between the i-th access point's position (x_i, y_i, z_i) and the receiver is $d_i = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}$, RSS vector $\mathbf{R}$ is given by

$$\mathbf{R} = (g(d_1), g(d_2), \cdots) \quad (2)$$

where g denotes a common function representing the relationship between RSS and distance. By trilateration, receiver positions are calculated by solving the following simultaneous equation with the observed RSS of each receiver.

$$d_i = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} = g^{-1}(\mathbf{R}) \quad (3)$$

B. Sampling based approach

In a real situation, RSS is effected by reflection waves caused by walls or obstacles. Trilateration cannot therefore achieve high positioning accuracy. To resolve this problem, a probabilistic model of RSS is created, using a set of previously measured RSS which includes the effect of reflection waves.

Roos et al.[7] applied a machine-learning framework to indoor positioning. In detail, positioning error of three machine-learning-based methods (NN, kernel, and histogram) were compared. The accuracies of the kernel and histogram methods exceeded that of the NN method by 25 to 30% in terms of average distance error. These technologies are used in an indoor-positioning system provided by Ekahau, Inc.[11]

Bahl et al.[5] presented experimental results concerning NN based indoor-positioning methods. In this experiment, a target was moved along a hall way in a building. Average distance error of positioning determined by the NN-based method was about 3 m. In the paper, a propagation modeling considering wall obstacles is additionally proposed, aiming to generate RSS samples. The propagation-modeling method is based on the "floor-attenuation-factor propagation model" (FAF)[12]. Parameters of the propagation model such as wall-attenuation factor are estimated by using measured RSS. To estimate wall-attenuation factors, the layout information of the building is used.

C. Contributions

As for indoor positioning, the reflection effect of RSS must be considered because there are a lot of obstacles indoors. Trilateration based approach cannot consider these effects. Propagation modeling proposed in [5] can only deal with a known number of walls as obstacles. As a result, the model cannot describe arbitrary obstacles (e.g., shelves or pillars) that are sometimes not marked on a map. In contrast, the probabilistic modeling can consider arbitrary obstacles because measured RSS includes obstacle effects, though gathering enough calibration samples is costly.

To overcome these problems, a technique called "RSS distribution modeling using the mirror-image method" (RDMMI), which is based on the mirror-image method for expressing reflective waves, is proposed here. By this method, RSS values at several points in a positioning area are measured in advance. Using these RSS values, RDMMI models RSS distribution according to radio-source positions. The experimental evaluation of RDMMI (described as follows) shows that this method describes multiple reflective waves and attains high positioning accuracy even if the calibration samples are few and non-uniformly placed. Furthermore, positions of access points can be estimated as model parameters. Even in this case, the proposed method achieved practicable positioning accuracy.

III. RDMMI positioning method

The process of the proposed method, which is called RDMMI, is as below. First, the models function $f_i(x, y, z; \Theta)$, which indicates RSS on position (x, y, z) from i-th radio source, by using the mirror-image method [13] to express reflective waves. Θ is a set of model parameters of f_i. Second, the model parameter set Θ is estimated using a maximum-likelihood method for fitting the function to measured RSS values. Finally, a receive point is estimated by maximizing posterior probability $P(x, y, z|\mathbf{R}, \Theta)$.

A. Mirror Image Method for RSS distribution modeling (RD-MMI)

In the step for estimating f_i, Θ that maximizes logarithmic posterior probability $\ln P(\Theta|\mathbf{C})$ is calculated, where $\mathbf{C} = \{(x_1, y_1, z_1, \mathbf{R}_1), (x_2, y_2, z_2, \mathbf{R}_2) \cdots (x_n, y_n, z_n, \mathbf{R}_n)\}$ is a vector of all the observation data from every access points. Here, (x_n, y_n, z_n) indicates n-th data measurement point, $\mathbf{R}_n$ indicates a vector of n-th observation data from each access point.

First, because of the independence of RSSs received from each access point, a model parameter set Θ_i which maximizes a posterior probability $P(\Theta_i|\mathbf{C})$ is estimated for each access point independently. $P(\Theta_i|\mathbf{C})$ can be expressed with probabilistic processes of each measurement data $P(\Theta_i|x_n, y_n, z_n, R_{n,i})$ as

$$\ln P(\Theta_i|\mathbf{C}) = \sum_n \ln P(\Theta_i|x_n, y_n, z_n, R_{n,i}) \quad (4)$$

where $R_{n,i}$ is a n-th measurement data from a i-th access point. According to the Bayes rule,

$$P(\Theta_i|x_n, y_n, z_n, R_{n,i}) \propto P(R_{n,i}|x_n, y_n, z_n, \Theta_i)P_i(\Theta_i) \quad (5)$$

It follows that,

$$\ln P(\Theta_i|\mathbf{C}) = \sum_n (\ln P(R_{n,i}|x_n, y_n, z_n, \Theta_i) + \ln P(\Theta_i)) + \text{const} \quad (6)$$

$P(R_{n,i}|x_n, y_n, z_n, \Theta_i)$ can be modeled assuming that $R_{n,i}$ obeys a Gaussian distribution with an average of $f_i(x_i, y_i, z_i; \Theta_i)$ and a variance σ. σ is assumed to be constant, for convenience; however, to be exact, parameter σ should be estimated. Here, $P(\Theta_i)$ is assumed to be a uniform distribution. Θ_i can therefore be estimated by using the least-square technique (e.g. the Newton's method [14] or an evolutionary algorithm [15]) to maximize the following likelihood function:

$$\ln P(\Theta_i|\mathbf{C}) = -\frac{1}{2\sigma^2} \sum_n (R_{n,i} - f_i(x_n, y_n, z_n; \Theta_i))^2 + \text{const} \quad (7)$$

The important point is how to formulate the function f_i. In our proposed method, the function f_i is modeled assuming that measured RSS is a sum of radio wave directly received from an access point (direct wave) and that reflected by walls or obstacles (reflective wave). An reflective wave is supposed to arrive from an imaginary source which is symmetrical to the position of the access point with respect to the reflecting surface. This is an approach based on the mirror image method [13] known in a field of electromagnetics, therefore the proposed method is called "mirror image method for RSS distribution modeling (RDMMI)". A conceptual diagram of the proposed method is shown in Fig. 2.

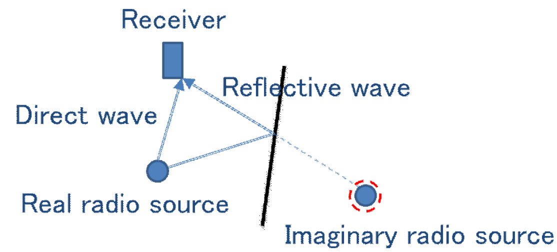

Figure 2. RSS modeling using the mirror image method

RSS is attenuated inversely related to a square of a distance between a radio source and a receiver. RSS is represented by the log scale unit (dBm), f_i can be formulated as,

$$f_i(x, y, z; \Theta_i) = 10 \log_{10} \sum_k \frac{b_{k,i}}{(\mathbf{r} - \mathbf{r_{k,i}})^{a_{k,i}}} \quad (8)$$

where $\mathbf{r} = (x, y, z)$. k is identifier for radio sources of direct and reflective waves, which is called dimensions of the function f_i. $\mathbf{r_{k,i}} = (x_{k,i}, y_{k,i}, z_{k,i})$ which indicates a position of radio source of k-th direct or reflective wave. The model parameter set Θ is composed of $a_{k,i}$, $b_{k,i}$ and $(x_{k,i}, y_{k,i}, z_{k,i})$. $a_{k,i}$ indicates an attenuation factor, and $b_{k,i}$ relates an output intensity of a radio wave. The number of dimensions k is required to be determined to an optimal value, however, it is a future work and not discussed here. Generally, with larger number of k, the function f_i becomes more complex and expected to describe more detailed real situation. Although, a large number of training data are necessary to estimate a large number of parameters.

Additionally, eq. 8 can be deformed as follows,

$$f_i(x, y, z; \Theta_i) = 10 \log_{10}\left(\frac{1}{(\mathbf{r} - \mathbf{r_{0,i}})^{a_{n,i}}} + \sum_k \frac{b'_{k,i}}{(\mathbf{r} - \mathbf{r_{k,i}})^{a_{k,i}}}\right) + c_i \quad (9)$$

where $b'_{k,i} = \frac{b_{k,i}}{b_{0,i}}$, $c_i = 10log_{10}b_{0,i}$. The purpose of this deformation is to handle output intensities of reflective waves in proportion to that of a direct wave. Besides, $\mathbf{r_0} = (x_{0,i}, y_{0,i}, z_{0,i})$ can be fixed to the position of the i-th access point, or estimated as unknown parameters. In evaluations below, both the results are described.

B. Location estimation

In the location-estimation step, using estimated model parameter set Θ, position (x, y, z) that maximizes logarithmic posterior probability $P(x, y, z|\mathbf{R}, \Theta)$ is calculated. $\mathbf{R}$ indicates a vector of received RSS from each access point. According to the Bayes rule, the following function is maximized:

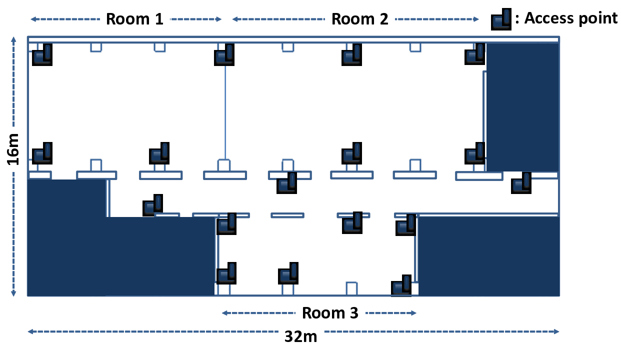

Figure 3. Points of access points. 17 access points are installed.

$$P(x,y,z|\mathbf{R},\Theta) \propto P(\mathbf{R}|x,y,z,\Theta)P(x,y,z) \tag{10}$$

$$\begin{aligned}&\ln P(x,y,z|\mathbf{R},\Theta)\\ &\quad= \ln P(\mathbf{R}|x,y,z,\Theta) + \ln P(x,y,z) + \text{const}\\ &\quad= \sum_i \ln P(R_i|x,y,z,\Theta_i) + \ln P(x,y,z) + \text{const}\end{aligned} \tag{11}$$

Here, $P(R_i|x,y,z,\Theta_i)$ can be modeled under the assumption that R_i obeys a Gaussian distribution with an average of $f_i(x,y,z;\Theta_i)$ and a variance σ as follows:

$$\begin{aligned}\ln P(x,y,z|\mathbf{R},\Theta) = &-\frac{1}{2\sigma^2}\sum_i (R_i - f_i(x,y,z;\Theta_i))^2\\ &+ \ln P(x,y,z) + \text{const}\end{aligned} \tag{12}$$

The distribution of position likelihood $P(x,y,z)$ can be assumed as a uniform distribution for the sake of ease. Finally, position (x,y,z) is calculated by minimizing $\sum_i (R_i - f_i(x,y,z;\Theta_i))^2$. The simplest way to minimize $\sum_i (R_i - f_i(x,y,z;\Theta_i))^2$ is determining a reference data with the minimum mean square distance. Reference data are generated on a grid point using the estimated RSS distribution model. Furthermore, on the purpose of estimating trajectories, the function $f_i(x,y,z;\Theta)$ can be applied to particle filters or Kalman Filters [16] to estimate a position (x,y,z).

IV. Experiment

A. Experimental settings

RDMMI with RSS data was evaluated in a experimental field with three rooms and a hall way. Figure 3 shows a map of the experimental area. The area was about 350 m^2. Walls and pillars are also shown in the figure. The rooms are separated by walls or a curtain (a boundary between the room 1 and 2). Each room has several openings (with no door). In this field, 17 access points (LAN-W150N/AP by logitec Corp.) were installed on the ground. Positions of the access points are shown in Fig. 3.

In this experimental field, RSS data are measured using android terminals on 110 sampling points. Sampling points of a training data set and a test data set, which are different among evaluation experiments, are described below. For each sampling point, averaged RSS of 30 measured data is used for evaluations.

Location-estimation errors due to RDMMI, NN and trilateration are compared in each evaluation. For NN, a sampling point of a training data which has the nearest RSS values to those of a test data is an estimated target position. For trilateration, test-data whose value is greater than -65 dBm are used to evaluate positioning error. Before the trilateration calculation, sets of three receivers that don't lie on a straight line (i.e., a sine of the angle which positions of the three receivers make is greater than 0.2) are chosen. The target position was, then, determined as a centroid of positions calculated by trilateration for each chosen set of receivers.

B. Average error comparison

Location-estimation error due to RDMMI was compared to NN and trilateration. For this evaluation, 17 access points, a training data set of 42 data and a test data set of 34 data were used. The sampling points of the data sets are shown in Fig. 4.First, using the training-data set, the RSS distribution model is estimated by RDMMI. Second, the estimated RSS of 1m-grid points are calculated by the RSS distribution model. Third, the positions of the test-data set are estimated by the NN method using the grid-point RSS.

As explained above, the RDMMI model describes reflective waves by the mirror method, and the number of functional dimensions indicates the level of complexity, that is, how many reflective waves are assumed. Accordingly, as the functional dimension becomes larger, the model becomes more complex. A complex model, however needs a huge amount of training data, because the number of parameters to be estimated is three times the number of dimensions. In this evaluation, a number of dimension is determined as a dimension between 0 (no reflective wave) to 15 which has the least error. Furthermore, a radio source of the first dimension of the model can either be estimated or fixed at the position of the access point. In this evaluation, both patterns are evaluated.

Figure 5 shows average errors due to RDMMI (AP position is known and unknown), NN and trilateration. Average errors are 3.1 m for RDMMI (AP positions unknown), 3.0 m for RDMMI (AP positions known), 3.7 m for NN and 8.7 m for trilateration. RDMMI reduces the positioning error by 16.2% (0.6 m) in comparison to that of NN.

C. Positioning error with a small training data set

One of the advantages of RDMMI is reduction of cost for sampling training data. In this evaluation, location-estimation errors with various number of training data are presented. Sampling points of training data (20, 83 points) are shown in Fig. 6. Sampling points of a training data set with 42 points and test data set are same as that shown in Fig. 4. For each training data pattern, a number of model dimension is determined as a dimension between 0 (no reflective wave) to 10 which has the least error. Figure 7 shows average errors

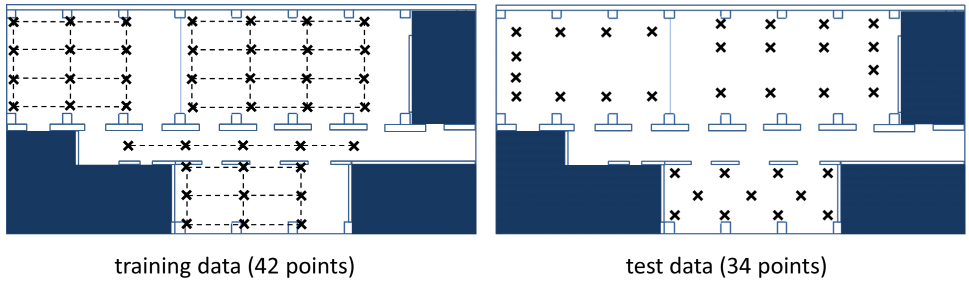

Figure 4. Sampling points of a training data set and a test data set

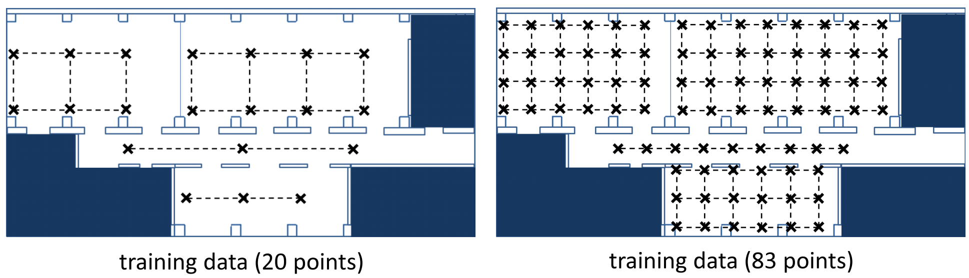

Figure 6. Sampling points of training data set

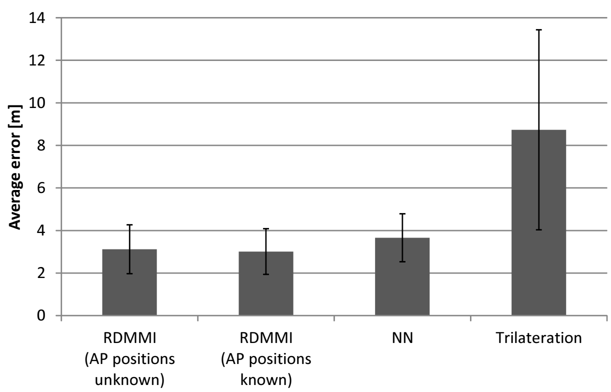

Figure 5. Error comparison

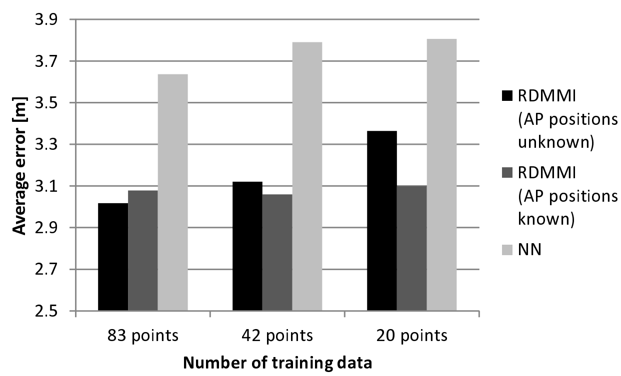

Figure 7. Error comparison according to the number of training data

due to RDMMI (AP position is known and unknown) and NN. Average errors of RDMMI (AP position unknown) are 3.4, 3.1, 3.0 m with 20, 42, 83 points of training data. In this case, though average error increases according to decreasing training data, RDMMI with 20 training data reduces the positioning error by 10.4% (0.4 m) in comparison to that of NN. For the case with known AP positions, average errors of RDMMI are about 3.1 m with all the training data patterns. In this case, RDMMI achieved practicable positioning accuracy with only 20 points of training data.

To evaluate a robustness of the proposed method with respect to choices of training data sets, positioning accuracy of RDMMI in the case that RSS measurements were randomly chosen was evaluated and compared with that of NN. For this evaluation, 20 sample RSS measurements were randomly selected from the measurement points shown in Fig. 6 (83 points). Ten randomly sampled data sets were used as training data to evaluate location-estimation errors due to RDMMI and NN. The number of model dimensions of RDMMI is determined as a dimension between 0 (no reflective wave) to 10 which has the least error. Figure 8 shows average errors due to RDMMI (AP positions are known and unknown) and NN. Average error are 3.7 m with 0.2 m of standard deviation for RDMMI (AP positions unknown), 3.3 m with 0.18 m of standard deviation for RDMMI (AP positions known), and 4.3 m with 0.16 m of standard deviation for NN. RDMMI achieved stably low positioning error no matter how to choose a training data set.

D. Positioning error with a small number of APs

In practical use, high positioning accuracy with small number of access points is beneficial. Therefore, positioning errors

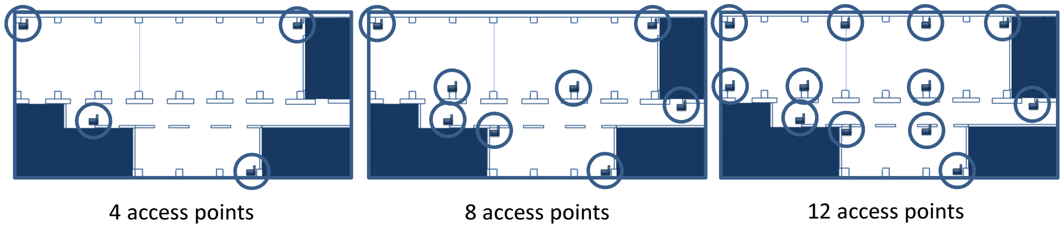

Figure 9. Positions of access points

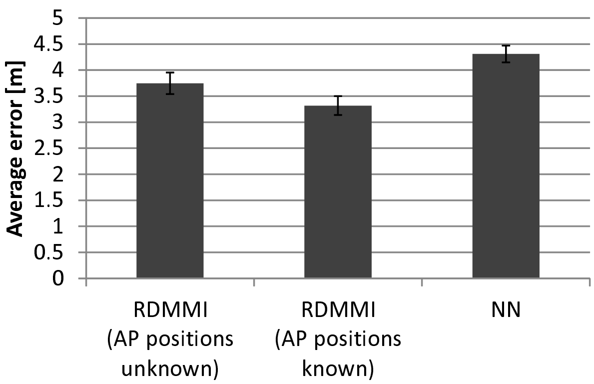

Figure 8. Error comparison with randomly chosen training data set

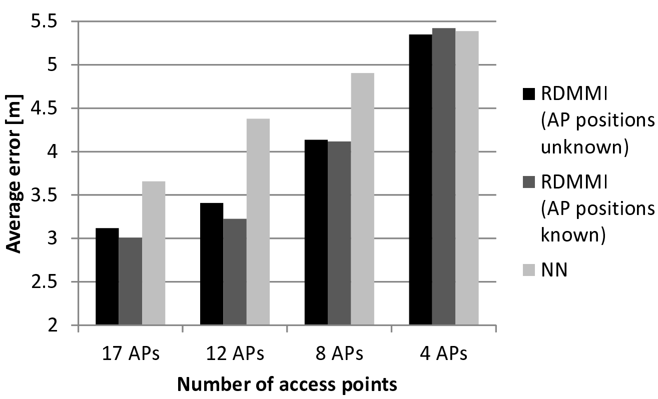

Figure 10. Error comparison according to the number of access points

with limited number of access points are presented in this section. Location-estimation errors with 4, 8, 12, 17 access points are evaluated. The positions of the access points are shown in Fig. 9. Positions of 17 access points are same as that shown in Fig. 3. Training data set and test data set are same as that shown in Fig. 4. A number of model dimension is determined as a dimension between 0 (no reflective wave) to 15 which has the least error.

Figure 10 shows average errors due to RDMMI and NN, according to numbers of access points. A number of access points affect to information amount of RSS vector, therefore a positioning error increases as a number of access points decreases. In the case of relatively big number of access points, the result shows that RDMMI achieves smaller average error than NN. However, with 4 access points, average errors of RDMMI and NN are almost comparable. This may caused by low-quality of the RSS distribution model. To specify the lowering factor and solve the problem is a future work.

V. Conclusion

A novel indoor-positioning method (called ”RSS distribution modeling using the mirror image method,” RDMMI), which uses RSS distribution modeling using the mirror method to consider the radio wave reflection effect, was developed and evaluated. The positioning accuracy of RDMMI was shown to exceed conventional NN and trilateration methods. Positioning accuracy with small sets of training data was evaluated. It was found that RDMMI achieves higher accuracy (3.4 m in the worst case with 20 training data) than NN. Furthermore, positioning accuracy in the case of a small number of access points was evaluated. The average error in the case with 17, 12, 8 access points was smaller than NN. However, the error in the case with 4 access points is comparable to NN method. These evaluation results shows that the proposed method can appropriately estimate an RSS distribution, while considering radio wave reflection, and achieve high positioning accuracy. In addition, even in the case that positions of access points are unknown, RDMMI achieved higher accuracy than NN.

As for future works, three tasks remain. First, the optimal model complexity (functional dimensions of the model) should be automatically determined. Second, the proposed method should be combined with a tracking method, such as a particle filter, to improve tracking accuracy. Third, to achieve higher positioning accuracy, radio wave noise should be considered. One approach for considering noise is using human body orientations, as mentioned by King et al.[17]

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