Comparison of QCLS Location Algorithms Using Two-Way Ranging Measurements

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***Abstract*—In case of the outdoor or indoor navigation, iterative Gauss-Newton method is apt to diverge especially at poor geometry. To avoid divergence, quadratic corrected least square (QCLS) method was introduced for one-way ranging (OWR) time-difference-of-arrival (TDOA) measurements. It is guaranteed that the solution by QCLS would not diverge with high positioning accuracy compatible to GN method. Compared to the OWR, two-way ranging (TWR) measurements contain no clock bias. This paper presents two types of QCLS methods for TWR measurements; one is QCLS with standard TWR measurements, the other is QCLS with differenced TWR measurements. By computer simulation, performance of two QCLS methods is compared to that of Gauss-Newton (GN) method.**

Keywords : QCLS; two-way ranging; localization

I. INTRODUCTION

The global navigation satellite system (GNSS) is widely used in computing user position in outdoor environment with an iterative algorithm such as Gauss-Newton (GN). For indoor location, WLAN or WPAN ranging systems including IR-UWB and CSS-UWB have been developed. When the GN method is applied to indoor location systems, however, positioning solution tends to diverge especially at poor dilution of precision (DOP) condition. [1] To avoid this problem, non-iterative location algorithms have been developed including Davidon-Fletcher-Powell (DFP), quadratic correction least square (QCLS), and linear correction least square (LCLS) method. [2-4]

Compared to the Gauss-Newton method, QCLS algorithm utilizes a dummy variable to linearize the measurement equation. Because the dummy variable is subject to other variables, location solution is obtained in two steps to refine the estimates. While one-way ranging (OWR) measurement contains clock bias, two-way ranging (TWR) measurement contains no bias. In case of TWR, measurement equation can be linearized in different manners. This paper presents two types of QCLS methods using TWR measurements. In the first approach, a dummy variable that is a function of user position is employed and a refined solution is obtained in two steps. The other approach utilizes differencing technique to linearize the measurement equation and the user position estimate is refined using undifferenced equations.

II. QCLS METHOD WITH TWR MEASUREMENTS

By squaring measurement equation and employing a dummy variable, measurement equation can be linearized. Because the dummy variable is a function of user position, user position estimate can be refined using quadratic correction in the second stage.

A. First step

The TWR measurements between the tag and the i-th anchor is given as

$$r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} + v_i \tag{1}$$

where x, y, z is tag position, x_i, y_i, z_i is position of the i-th anchor. v_i is assumed to be a white Gaussian noise with variance σ^2. Squaring (1), the equation can be rewritten as

$$\begin{aligned} r_i^2 &= x^2 - 2x\cdot x_i + x_i^2 + y^2 - 2y\cdot y_i + y_i^2 \\ &+ z^2 - 2z\cdot z_i + z_i^2 + 2d_i v_i + v_i^2 \end{aligned} \tag{2}$$

where $d_i = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}$. By defining dummy variable $K = x^2 + y^2 + z^2$, (2) becomes

$$\begin{aligned} r_i^2 &= K - 2x\cdot x_i + x_i^2 - 2y\cdot y_i + y_i^2 \\ &- 2z\cdot z_i + z_i^2 + 2d_i v_i + v_i^2 \end{aligned} \quad \text{.......(3)}$$

With n range measurements, measurement equation is written in a linear matrix-vector form that is given by

$$A\underline{x} = \underline{b} + \underline{w} \tag{4}$$

where $A = \begin{bmatrix} 2x_1 & 2y_1 & 2z_1 & -1 \\ 2x_2 & 2y_2 & 2z_2 & -1 \\ \vdots & \vdots & \vdots & -1 \\ 2x_n & 2y_n & 2z_n & -1 \end{bmatrix}$, $\underline{x} = \begin{bmatrix} x \\ y \\ z \\ K \end{bmatrix}$,

$\underline{b} = \begin{bmatrix} x_1^2 + y_1^2 + z_1^2 - r_1^2 \\ x_2^2 + y_2^2 + z_2^2 - r_2^2 \\ \vdots \\ x_n^2 + y_n^2 + z_n^2 - r_n^2 \end{bmatrix}$ and $\underline{w} = \begin{bmatrix} 2d_1v_1 + v_1^2 \\ 2d_2v_2 + v_2^2 \\ \vdots \\ 2d_nv_n + v_n^2 \end{bmatrix}$.

The covariance matrix of the noise $\underline{w}$ is given by

$$Q_w = \begin{bmatrix} 4d_1^2\sigma^2 & 0 & \cdots & 0 \\ 0 & 4d_2^2\sigma^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 4d_n^2\sigma^2 \end{bmatrix} + \begin{bmatrix} 2\sigma^4 & 0 & \cdots & 0 \\ 0 & 2\sigma^4 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2\sigma^4 \end{bmatrix} \quad (5)$$

where σ^2 is the variance of the ranging measurement and d_i is true distance between the tag and the i-th anchor. Using (4) and (5), the position estimate is obtained using weighted least-square that is given by

$$\hat{\underline{x}} = (A^T Q_w^{-1} A)^{-1} A^T Q_w^{-1} \underline{b} \quad (6)$$

Because d_i is unknown, d_i is replaced by the range measurement r_i in estimating $\hat{\underline{x}}$.

B. *Second Step*

Because K is a function of user position, relationship between the four estimates are used in the second step. Defining $\hat{\underline{x}} - \underline{x} = \tilde{\underline{x}}$ where $\tilde{\underline{x}} = [\tilde{x} \quad \tilde{y} \quad \tilde{z} \quad \tilde{K}]^T$ and squaring its elements, the errors of the squared variables are written as

$$\begin{aligned} \hat{x}^2 &= x^2 + 2x\cdot\tilde{x} + \tilde{x}^2 \\ \hat{y}^2 &= y^2 + 2y\cdot\tilde{y} + \tilde{y}^2 \\ \hat{z}^2 &= z^2 + 2z\cdot\tilde{z} + \tilde{z}^2 \\ \hat{K} &= K + \tilde{K} \end{aligned} \quad (7)$$

If the error $\tilde{\underline{x}}$ is sufficiently small, the second order error terms are negligible. Then, the relationship between true position and estimates is approximated as

$$C\underline{x}_s \cong \underline{d} - \underline{w}' \quad (8)$$

where $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, $\underline{x}_s = \begin{bmatrix} x^2 \\ y^2 \\ z^2 \end{bmatrix}$, $\underline{d} = \begin{bmatrix} \hat{x}^2 \\ \hat{y}^2 \\ \hat{z}^2 \\ \hat{K} \end{bmatrix}$, and $\underline{w}' = \begin{bmatrix} 2x\cdot\tilde{x} \\ 2y\cdot\tilde{y} \\ 2z\cdot\tilde{z} \\ \tilde{K} \end{bmatrix}$.

Because the error covariance matrix of $\hat{\underline{x}}$ is given by $(A^T Q_w^{-1} A)^{-1}$, the covariance matrix of $\underline{w}'$ is written as

$$Q_{w'} = P Q_{\hat{x}} P^T \quad (9)$$

where $Q_{\hat{x}} = (A^T Q_w^{-1} A)^{-1}$ and $P = \begin{bmatrix} 2x & 0 & 0 & 0 \\ 0 & 2y & 0 & 0 \\ 0 & 0 & 2z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Using (8) and (9), the refined estimate is obtained by

$$\hat{\underline{x}}_s = (C^T Q_{w'}^{-1} C)^{-1} C^T Q_{w'}^{-1} \underline{d} \quad (10)$$

Because user position in P is unknown, the estimated value in (6) is used instead. From $\hat{\underline{x}}_s$, corrected user position can be obtained.

III. QCLS METHOD WITH DIFFERENCED TWR MEASUREMENTS

By differencing the squared measurement equation, measurement equation can be linearized. In this case, the dimension of differenced measurement vector becomes (n-1), which means that the measurements are projected to differenced space to partially lose information in the original measurement space. Hence, position correction has to be made using undifferenced ranging measurements.

A. *First step*

Supposing the first measurement as the reference, referential differencing with squared measurements leads to [5]

$$\begin{aligned} & r_1^2 - r_i^2 = \\ & K - 2x\cdot x_1 + x_1^2 - 2y\cdot y_1 + y_1^2 - 2z\cdot z_1 + z_1^2 + 2d_1 v_1 + v_1^2 \\ & -(K - 2x\cdot x_i + x_i^2 - 2y\cdot y_i + y_i^2 - 2z\cdot z_i + z_i^2 + 2d_i v_i + v_i^2) \end{aligned} \quad (11)$$

Rearranging (11), it is written as

$$\begin{aligned} & (x_1 - x_i)x + (y_1 - y_i)y + (z_1 - z_i)z \\ & = \frac{1}{2}(x_1^2 - x_i^2 + y_1^2 - y_i^2 + z_1^2 - z_i^2 - r_1^2 + r_i^2) + w_{d(i)} \end{aligned} \quad (12)$$

where $w_{d(i)} = \frac{1}{2}(2d_1 v_1 - 2d_i v_i + v_1^2 - v_i^2)$. With n range measurements, (12) is written as a linear matrix-vector form that is given by

$$A_d \underline{x}_d = \underline{b}_d + \underline{w}_d \quad (13)$$

where

$$A_d = \begin{bmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ x_1 - x_3 & y_1 - y_3 & z_1 - z_3 \\ \vdots & \vdots & \vdots \\ x_1 - x_n & y_1 - y_n & z_1 - z_n \end{bmatrix}, \quad \underline{x}_d = \begin{bmatrix} x \\ y \\ z \end{bmatrix},$$

$$\underline{b}_d = \frac{1}{2}\begin{bmatrix} x_1^2 - x_2^2 + y_1^2 - y_2^2 + z_1^2 - z_2^2 - r_1^2 + r_2^2 \\ x_1^2 - x_3^2 + y_1^2 - y_3^2 + z_1^2 - z_3^2 - r_1^2 + r_3^2 \\ \vdots \\ x_1^2 - x_n^2 + y_1^2 - y_n^2 + z_1^2 - z_n^2 - r_1^2 + r_n^2 \end{bmatrix}$$

and $\underline{w}_d$ is the vector of $w_{d(i)}$. Note that the dimension of the vector $\underline{b}_d$ is n-1. The covariance matrix of $\underline{w}_d$ is given by

$$\begin{aligned} Q_{w_d} = \sigma^2 & \begin{bmatrix} d_1^2 + d_2^2 & d_1^2 & \cdots & d_1^2 \\ d_1^2 & d_1^2 + d_3^2 & \cdots & d_1^2 \\ \vdots & \vdots & \ddots & \vdots \\ d_1^2 & d_1^2 & \cdots & d_1^2 + d_n^2 \end{bmatrix} \\ & + \frac{\sigma^4}{2}\begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 2 \end{bmatrix} \end{aligned} \quad (14)$$

Using (13) and (14), user position estimate is obtained by

$$\hat{\underline{x}}_d = (A_d^T Q_{w_d}^{-1} A_d)^{-1} A_d^T Q_{w_d}^{-1} \underline{b}_d \quad (15)$$

B. *Second step*

In the second step, correction is made using undifferenced TWR measurements. Using the relationship $\hat{\underline{x}}_d = \underline{x}_d + \tilde{\underline{x}}_d$

where $\underline{\tilde{x}}_d = [\tilde{x}_d \quad \tilde{y}_d \quad \tilde{z}_d]^T$, the errors in position estimate and i-th measurement are represented as

$$\begin{aligned}(\hat{x}-x_i)^2 &= (x-x_i)^2 + 2(x-x_i)\cdot\tilde{x}_d + \tilde{x}_d^{\,2}\\ (\hat{y}-y_i)^2 &= (y-y_i)^2 + 2(y-y_i)\cdot\tilde{y}_d + \tilde{y}_d^{\,2}\\ (\hat{z}-z_i)^2 &= (z-z_i)^2 + 2(z-z_i)\cdot\tilde{z}_d + \tilde{z}_d^{\,2}\\ r_i^{\,2} &= (x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2 - 2r_i\cdot v_i + v_i^{\,2}\end{aligned} \tag{16}$$

Assuming that $\underline{\tilde{x}}_d$ and v_i are sufficiently small, it can be approximated as

$$C\underline{x}_{s(i)} \cong \underline{d}_{d(i)} - \underline{w}_{d(i)}' \tag{17}$$

where $C = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\1&1&1\end{bmatrix}$, $x_{s(i)} = \begin{bmatrix}(x-x_i)^2\\(y-y_i)^2\\(z-z_i)^2\end{bmatrix}$,

$\underline{d}_{d(i)} = \begin{bmatrix}(\hat{x}-x_i)^2\\(\hat{y}-y_i)^2\\(\hat{z}-z_i)^2\\r_i^{\,2}\end{bmatrix}$ and $\underline{w}_{d(i)}' = \begin{bmatrix}2(x-x_i)\cdot\tilde{x}_d\\2(y-y_i)\cdot\tilde{y}_d\\2(z-z_i)\cdot\tilde{z}_d\\2r_i\cdot v_i\end{bmatrix}$.

The covariance matrix of $\underline{w}_{d(i)}'$ is given by

$$Q_{w_{d(i)}'} \cong P_{d(i)}\begin{bmatrix}Q_{\hat{x}_d} & \underline{0}\\ \underline{0} & \sigma^2\end{bmatrix}P_{d(i)}^{\,T} \tag{18}$$

where $Q_{\hat{x}_d} = (A_d^{\,T}Q_{w_d}^{\,-1}A_d)^{-1}$ and

$$P_{d(i)} = \begin{bmatrix}2(x-x_i)&0&0&0\\0&2(y-y_i)&0&0\\0&0&2(z-z_i)&0\\0&0&0&2r_i\end{bmatrix}.$$

To calculate $P_{d(i)}$, the position estimate $\underline{\hat{x}}_d$ in (15) is used. Using (17) and (18), $\underline{\hat{x}}_{s(i)}$ is solved as

$$\underline{\hat{x}}_{s(i)} = (C^T Q_{w_{d(i)}'}^{\,-1}C)^{-1}C^T Q_{w_{d(i)}'}^{\,-1}\underline{d}_{d(i)} \tag{19}$$

The error covariance matrix of $\underline{\hat{x}}_{s(i)}$ is given by $Q_{\underline{\hat{x}}_{s(i)}} = (C^T Q_{w_{d(i)}'}^{\,-1}C)^{-1}$. The corrected position estimate using i-th measurement is represented as

$$\underline{\hat{x}}^c_{d(i)} = \begin{bmatrix}\hat{x}^c_{d(i)}\\ \hat{y}^c_{d(i)}\\ \hat{z}^c_{d(i)}\end{bmatrix} = \begin{bmatrix}x_i \pm\sqrt{\hat{x}_{s(i)}}\\ y_i \pm\sqrt{\hat{y}_{s(i)}}\\ z_i \pm\sqrt{\hat{z}_{s(i)}}\end{bmatrix} \tag{20}$$

Using (15) and information of the workspace, sign ambiguity in (20) can be resolved. Assuming that $\underline{\hat{x}}_{s(i)} = \underline{x}_{s(i)} + \underline{\tilde{x}}_{s(i)}$ and $\underline{\tilde{x}}_{s(i)} = [\tilde{x}_{s(i)} \quad \tilde{y}_{s(i)}, \quad \tilde{z}_{s(i)}]^T$, the errors in $\underline{\hat{x}}^c_{d(i)}$ are approximated as

$$\begin{aligned}\tilde{x}^c_{d(i)} &= \sqrt{\hat{x}_{s(i)}} - \sqrt{x_{s(i)}} \cong \frac{1}{2}\frac{\tilde{x}_{s(i)}}{\sqrt{x_{s(i)}}}\\ \tilde{y}^c_{d(i)} &= \sqrt{\hat{y}_{s(i)}} - \sqrt{y_{s(i)}} \cong \frac{1}{2}\frac{\tilde{y}_{s(i)}}{\sqrt{y_{s(i)}}}\\ \tilde{z}^c_{d(i)} &= \sqrt{\hat{z}_{s(i)}} - \sqrt{z_{s(i)}} \cong \frac{1}{2}\frac{\tilde{z}_{s(i)}}{\sqrt{z_{s(i)}}}\end{aligned} \tag{21}$$

If $\frac{\tilde{x}_{s(i)}}{\sqrt{x_{s(i)}}}$, $\frac{\tilde{y}_{s(i)}}{\sqrt{y_{s(i)}}}$ and $\frac{\tilde{z}_{s(i)}}{\sqrt{z_{s(i)}}}$ are sufficiently small, the error covariance matrix of $\underline{\hat{x}}^c_{d(i)}$ is written as

$$Q_{\hat{x}^c_{d(i)}} = P_{s(i)}Q_{x_{s(i)}}P_{s(i)}^{\,T} \tag{22}$$

where $P_{s(i)} = \begin{bmatrix}\frac{1}{2|x-x_i|}&0&0\\0&\frac{1}{2|y-y_i|}&0\\0&0&\frac{1}{2|z-z_i|}\end{bmatrix}$.

Using n-dimensional range measurements, (n×3) corrected position estimates can be obtained which is given by

$$\underline{\hat{x}}_T = Hx + \underline{w}_T \tag{23}$$

where $H = \begin{bmatrix}I_3\\I_3\\\vdots\\I_3\end{bmatrix}$, $\underline{\hat{x}}_T = \begin{bmatrix}\underline{\hat{x}}^c_{d(1)}\\\underline{\hat{x}}^c_{d(2)}\\\vdots\\\underline{\hat{x}}^c_{d(n)}\end{bmatrix}$, and $\underline{w}_T = \begin{bmatrix}\underline{e}_{\hat{x}^c_{d(1)}}\\\underline{e}_{\hat{x}^c_{d(2)}}\\\vdots\\\underline{e}_{\hat{x}^c_{d(n)}}\end{bmatrix}$.

Note that the dimension of the matrix H is (3n×3). From (23), the final position estimate is solved as

$$\underline{\hat{x}}^c = (H^T Q_{x_T}^{\,-1}H)^{-1}H^T Q_{x_T}^{\,-1}\underline{\hat{x}}_T \tag{24}$$

where $Q_{x_T} = E\{\underline{w}_T\cdot\underline{w}_T^{\,T}\}$.

IV. SIMULATION RESULTS

To evaluate those methods, two QCLS algorithm are compared with Gauss-Newton method. Anchors are set at (0, 0), (10, 0), (0, 10), and (10, 10). The standard deviation of noise is set to be 0.1m based on the experimental data. The root mean square error in 2D is calculated and averaged ten-thousand times. The two-way ranging QCLS algorithm shows similar rms error distribution which shows in Fig. 1.

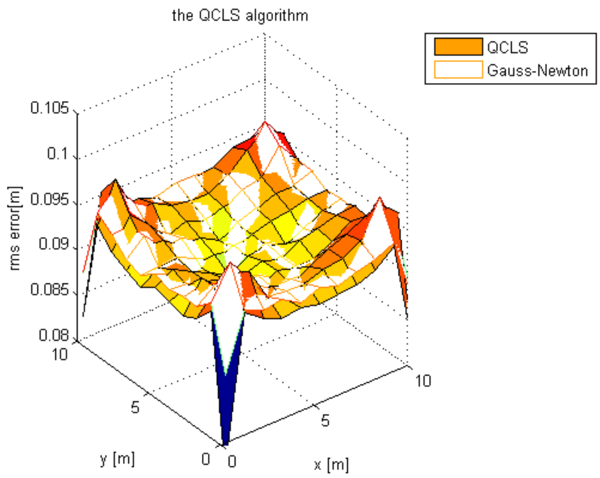

Figure 1 rms error of QCLS algorithm

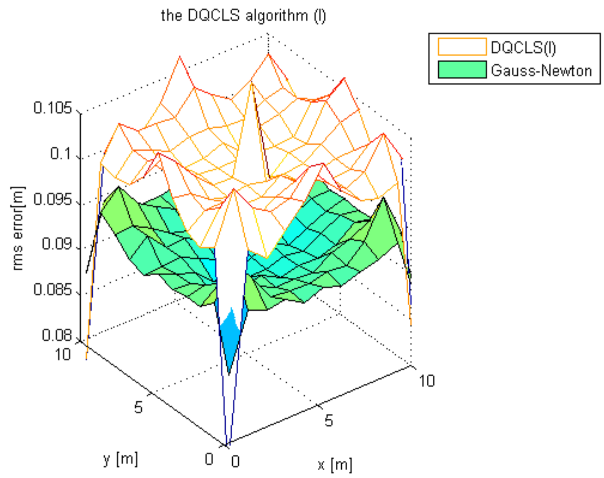

Figure 3 DQCLS algorithm (corrected by a measurement)

To analyzing performance of the two-way ranging differential QCLS method, this paper compares the solution before and after correction and also compares between correction method using one measurement and using all measurements. In Fig. 2, DQCLS algorithm with no correction method is shown.

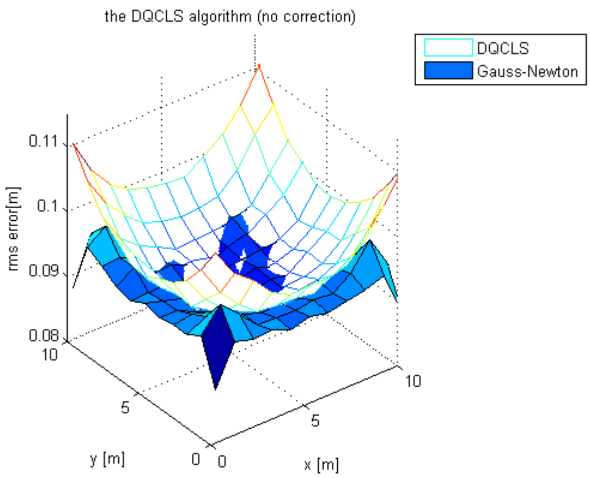

Figure 2 DQCLS algorithm (no correction)

The rms error is small at the center of the geometry. As a tag goes to the edge of the geometry, the rms error is getting greater. This solution needs to be corrected in proper manner. The first algorithm uses the measurement from the anchor which is the nearest from the position which is acquired in the first step. And the second algorithm utilizes all measurements. To begin with, first DQCLS algorithm is shown in Fig. 3.

Correcting by the shortest measurement, the rms error is alleviated at the edge of the geometry. However, this method shows higher rms error in the most part of the region compared to Gauss-Newton method. Then, the result of the second DQCLS algorithm is shown in Fig. 4. In this case, the edge of the geometry is excluded since the singularity problem due to (22) is occurred at those points.

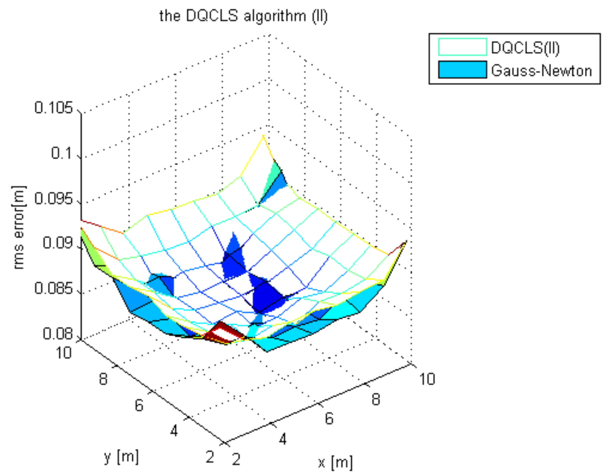

Figure 4 DQCLS algorithm (corrected by all measurements)

Compared to Fig. 3, overall performance is improved in the overall region to fit under a half of centimeter compared with the GN method. However, there is the singularity problem at the edge of the geometry. This would be further consideration.

V. Conclusion

This paper suggests DQCLS algorithms which uses every measurement to correct position. Compared to the existing QCLS method, DQCLS method differences squared equation in order to eliminate higher order unknowns. On the other hand, incomplete modeling of the covariance matrix of the equation causes degradation of the solution, which needs to improve it by correction method. The DQCLS algorithm using a measurement for position correction shows poor performance at the most of the geometry than Gauss-Newton method. The suggested DQCLS algorithm shows better performance than the DQCLS algorithm which uses a measurement and represents similar performance to Gauss-Newton method.

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