# Deeply Coupled GPS/INS Integration in Pedestrian Navigation Systems in Weak Signal Conditions

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Abstract—This paper describes non-coherent Deeply Coupled GPS/INS integration in a pedestrian navigation system to improve position accuracy and availability in weak signal conditions. A pedestrian navigation system consists of several sensors to calculate a position of a person to guide for example rescue missions. The system presented in this paper consists of a torso mounted IMU and is used for step detection and step length and heading estimation. Additionally a barometer, magnetometer and a GPS sensor for absolute positioning are used. Since pedestrian navigation systems often are used in challenging environments like urban canyons or indoors, the use of GPS signals is often restricted. We will show that by using a Deeply Coupled GPS/INS integration system, tracking of GPS signals under weak signal conditions is possible and a seamless transition between Indoor and outdoor situations is achieved. By applying the information of a position displacement between two steps from the step length and heading estimation GPS tracking and position accuracy can be increased.

For an optimal performance the system uses a deeply acquisition and re-acquisition routine. Therefore additional satellites can be used which could not have been acquired before, due to low signal to noise ratios. By carefully weighting the GPS measurements accordingly to their  $C/N_0$  and having a larger set of satellites available, position accuracy is increased compared to a non-vector tracking approach. The sensor fusion itself is realized in an error state space kalman filter and the step length update is performed using a state cloning technique preserving realistic position uncertainties in the filter.

With this approach tracking and acquisition of GPS signals inside buildings with  $C/N_0$  below 20dBHz is possible. In this paper we will show that using deep integration in GPS signal tracking including step length estimations increases position accuracy of a pedestrian navigation system and availability of GPS position updates

### I. INTRODUCTION

A Deeply Coupled GPS/INS System can be described as a coupling of signal tracking of single GPS receiver channels through a position, velocity and timing solution with simultaneous support of the navigation solution with dynamic information of an inertial navigation system. The signal tracking of each satellite benefits on below the other and is improved by the complementary characteristics of inertial sensors. Complementary characteristics are detection of highly dynamic movements, continuous availability and low vulnerability against interference. Looking at a pedestrian navigation system especially positioning under weak signal conditions and interference scenarios is of interest. Furthermore a short GPS reacquisition time is a crucial factor in urban environments as the number of satellites in view and therefore the available satellite constellation changes often.

The system presented in this paper is capable of using GPS measurements under weak signal conditions and makes possible a seamless transition between indoor and outdoor areas. To achieve a smooth transition the GPS measurements have to be weighted carefully by a robust carrier to noise  $(C/N_0)$  estimation.

In this work we will describe how a Deeply Coupled GPS/INS System can be extended by using pedestrian dead reckoning to further improve the navigation solution. This information is generated by a torso mounted pedestrian navigation system: A torso-mounted inertial measurement unit (IMU) is used for step detection and step length estimation and in combination with a magnetic sensor, heading is calculated. The combination of heading and step length yields a 2D polygon which can be used in a navigation filter as a delta position measurement, sometimes called step length update (SLU). The sensor fusion is realized by an error state kalman filter (EKF) and for an adequate processing of the step length updates a state cloning technique is used.

In the first part (section II) of this paper the Deeply Coupled GPS/INS System is described. In section III the Deeply Coupled GPS/INS System is extended to a Deeply Coupled GPS/PN System (Pedestrian Navigation) using SLUs. In section IV implementation details are described, including the software-defined-radio (SDR) GNSS receiver platform, SLU estimation and deeply acquisition and re-acquisition of GPS signals. In the last section the results of this work are shown.

#### II. DEEPLY COUPLED GPS/INS

When using a GPS receiver position and velocity information are available, but the attitude is unknown (single GPS receiver). By combining GPS and IMU the attitude in the system model is observable and attitude errors can be corrected. Additionally continuity and availability is improved. There are improved receiver technologies called high sensitivity receivers which are capable of calculating position solutions under weak signal conditions or even indoors, but the estimated positions are often inaccurate. Besides achieving high sensitivity under weak signal conditions results in low dynamic of the position solution due to high filtering. By increasing the receiver dynamic the position solution gets



Fig. 1. non-coherent Deeply Coupled GPS/INS System

less robust and continuous positioning under weak signal conditions becomes more difficult. In contrast to this an IMU has usually a update rate of 100Hz to 1kHz. Therefore an identification of high dynamic movements and a bridging of GPS outages of a few seconds to several minutes is possible.

## A. Non-Coherent Deeply Coupled GPS/INS

In Deeply Coupled (also called Ultra-Tightly) Systems the in-phase and quadrature-phase correlator outputs of a GPS receiver are processed in a inertial navigation system and simultaneously the signal tracking is performed by using the output of the navigation solution. This way tracking and positioning are forming a closed loop. The Deeply Coupled System in this work is realized as a non-coherent system, where the discriminator outputs are used directly to determine code phase and carrier frequency error. The carrier phase is not controlled. A non-coherent approach is more robust when  $C/N_0$  is low making it suitable for a pedestrian navigation system and indoor positioning. In this section the system model is described, an overview can be seen in Fig. 1.

The navigation filter is designed as a closed loop error state space kalman filter. The kalman filter estimates the error states of the strap-down algorithm (SDA) so that the absolute states of the SDA can be corrected. The kalman filter consist of the following 17 states

$$\Delta \hat{x} = (\Delta p^T, \Delta v^T, \Delta \psi^T, \Delta b_a^T, \Delta b_\omega^T, \Delta c_0 \delta t_{rc}^T)^T.$$
(1)

 $\Delta p^T$  and  $\Delta v^T$  are the estimated position and velocity errors,  $\Delta \psi$  are the attitude errors in Euler angles.  $\Delta b_a$  and  $\Delta b_{\omega}$  include the acceleration and rate bias of the inertial sensors.  $\delta t_{rc}$ consists of the clock error and the clock error drift estimation. The output of the navigation solution are the absolute states of the SDA as displayed in Fig. 1. During measurement updates prediction and measurement are compared using  $\Delta \tilde{y} = \hat{y}^{-} - \tilde{y}$ . The discriminator values used in the measurement step are available at the end of a navigation bit (20ms integration time is used). Because of different delays for each satellite these values are available asynchronous only. Creation of range measurements (tic-events) are performed for all satellites at the same time k. A time index k all lastly available discriminator measurements are processed. The asynchronism is neglected. Range measurements are performed at a tic-rate of 50Hz using the kalman filter equations. The measurement is processed for every satellite separately, avoiding a Matrix inversion to

calculate the kalman gain matrix  $K_k$ .

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}$$
(2)

$$\Delta \hat{x}_k^+ = \Delta \hat{x}_k^- - K_k (H_k \Delta \hat{x}_k^- - \Delta \tilde{y}) \tag{3}$$

$$P_{k}^{+} = (I - K_{k}H_{k})P_{k}^{-}$$
(4)

After processing all measurements the absolute states of the SDA are corrected with  $\hat{x}_k^+ = \hat{x}_k^- - \Delta \hat{x}_k^+$  and the kalman error state  $\Delta \hat{x}_k^+$  is set to zero.

1) code measurement: Before raw pseudo range measurements can be processed corrections for ionospheric-, tropospheric- and satellite clock errors have to be applied. The corrected measurements  $\tilde{\rho}_{rep}$  are compared to the predicted pseudo range  $\hat{\rho}^-$  of the kalman filter including the receiver clock error  $c_0 \delta \hat{t}_{rec}^-$ .

$$\hat{\rho}^- = |r_{sat} - r_{rec}| + c_0 \delta \hat{t}_{rec}^- \tag{5}$$

In a Deeply Coupled System the code phase error is included in the range measurement by using the code discriminator. The code phase error is the difference between the pseudo range  $\tilde{\rho}_{rep}$  extracted from the replica signal and the true pseudo range  $\rho$ . The code phase error  $\delta \tilde{x}$  measured by the code discriminator including noise  $\eta_{\delta x}$  is

$$\delta \tilde{x} = (\tilde{\rho}_{rep} - \rho) \frac{f_c o}{c_0} + \eta_{\delta x}.$$
 (6)

The measurement for the true pseudo range can be described as

$$\tilde{\rho} = \tilde{\rho}_{rep} - \delta \tilde{x} \frac{c_0}{f_c o}.$$
(7)

The residuum  $\Delta y_{psr}$  is used to process the measurement in the kalman filter. The measurement has to be weighted using an estimation of the discriminator variance  $\sigma^2_{D_{ELP,C/N_0}}$  utilizing the  $C/N_0$  estimation in section IV.

$$\Delta y_{psr} = \hat{\rho}^- - \tilde{\rho}_{rep} + \delta \tilde{x} \frac{c_0}{f_{co}}.$$
(8)

2) carrier measurement: The procedure for the carrier measurement is similar to the code measurement. The predicted range rate (doppler measurement)  $\hat{\rho}$  has to be corrected of the receiver clock error drift  $c_0 \delta \hat{t}_{rec}$ .

$$\hat{\dot{\rho}}^{-} = \mathbf{e}^{T} (\mathbf{v}_{sat} - \mathbf{v}_{rec}) + c_0 \delta \hat{\dot{t}}_{rec}^{-}$$
(9)

 $\mathbf{e}^T$  is the unity vector from receiver to satellite and  $\mathbf{v}_{sat}, \mathbf{v}_{rec}$  satellite and receiver velocity. Again the frequency discriminator is used in the carrier frequency measurement. The carrier frequency error  $\delta \tilde{f}_{ca}$  is the difference between the range rate from the replica signal  $\tilde{\rho}_{rep}$  and the true range rate  $\dot{\rho}$ .

$$\delta \tilde{f}_{ca} = (\tilde{\dot{\rho}}_{rep} - \dot{\rho}) \frac{f_{ca}}{c_0} + \eta_{\delta f_{ca}}$$
(10)

 $\eta_{\delta f_{ca}}$  is the noise of the discriminator. A measurement including the frequency discriminator for the true range rate  $\tilde{\rho}$  is

$$\tilde{\dot{\rho}} = \tilde{\dot{\rho}}_{rep} - \delta \tilde{f}_{ca} \frac{c_0}{f_{ca}}.$$
(11)

With equation 9 and 11 the kalman filter residuum  $\Delta y_{rr}$  can be calculated

$$\Delta y_{rr} = \hat{\dot{\rho}}^{-} - \tilde{\dot{\rho}}_{rep} + \delta \tilde{f}_{ca} \frac{c_0}{f_{ca}}.$$
 (12)

3) Gating (discard measurements): Because of the direct feedback of the navigation solution of the kalman filter to the signal tracking of the GPS receiver the measurements have do be checked for faulty values. As said before measuring the  $C/N_0$  is important resulting in a discriminator variance  $\sigma_D$  to weight the measurements. But especially under weak signal conditions as in urban canyons the a priori residuum can take very high or low values for a short period of time. This can be due to strong signal strength fluctuations, strong multi path propagation or receiver internal problems. To prevent these measurements from affecting the navigation solution a tolerance band for the a priori residuum is defined. The tolerance band s includes the variance of the measurement and the variance of the navigation solution. It is defined as three times standard deviation of the a priori residuum.

$$s = \pm 3 \cdot \sqrt{\mathbf{H} \mathbf{P}_{XX} \mathbf{H}^T + \sigma_D^2}.$$
 (13)

The tolerance band increases with increasing discriminator variance  $\sigma_D^2$  or with increasing uncertainty of the state estimation  $\mathbf{P}_{XX}$  depending of the satellite constellation accounted through the measurement matrix **H**.

#### III. DEEPLY COUPLED GPS/PN

In the last section the design of a Deeply Coupled GPS/INS was described which has been further developed to a Deeply Coupled GPS/PN System. The quality of the navigation solution of a Deeply Coupled GPS/INS System under weak signal conditions heavily depends on the quality of the IMU. Since the MEMS IMUs used in most pedestrian navigation systems are not capable of bridging long GPS outages the results are not satisfying. Therefore the system is supplemented by a step length estimation of a pedestrian navigation system, which is composed of a torso mounted IMU and a magnetometer. The IMU is used for step detection and step length estimation and in combination with the magnetometer the heading of the steps is estimated. See IV-B for further details. In the present section processing of step length estimation is described. To preserve a realistic variance of the position solution a stochastic cloning filter is introduced. In section V the results are additionally compared to a vector receiver using a constant velocity model without inertial aiding. This is the simplest form of deep integration.

## A. Processing of step length estimations and state cloning

To process the relative change in position a stochastic cloning kalman filter is implemented [1]. A stochastic cloning filter can be used where a measurement refers to a relative change of a state in a certain time interval. This concept for processing step length estimation is presented in [1] and is used to implement step length estimation in the Deeply Coupled GPS/INS System. Stochastic cloning is performed



Fig. 2. Measured and predicted position displacement with stochastic cloning

for the two absolute states of the strap-down latitude  $\varphi$  and longitude  $\lambda$ . After every step length update (SLU) these states are cloned ( $\varphi_c, \lambda_c$ ). The step length module provides a step angle  $\psi_s$  and a step length estimation  $l_s$  which can be translated into a  $\Delta north_c$  and  $\Delta east_c$  component. The position displacement  $\tilde{y}$  in ned-coordinates is

$$\tilde{y} = \begin{pmatrix} \Delta \tilde{n} \\ \Delta \tilde{e} \end{pmatrix} = \begin{pmatrix} l_s \cdot \cos(\psi_s) \\ l_s \cdot \sin(\psi_s) \end{pmatrix}$$
(14)

The prediction is performed trough the difference between cloned and actual absolute strap-down state by transforming the LLH to ned coordinates [2]. This is displayed in Fig. 2.

$$\hat{y} = \begin{pmatrix} \Delta \hat{n} \\ \Delta \hat{e} \end{pmatrix} \approx \begin{pmatrix} (\hat{\varphi}^- - \hat{\varphi}_c^-)(R_N(\hat{\varphi}^-) + \hat{h}^-) \\ (\hat{\lambda}^- - \hat{\lambda}_c^-)((R_E(\hat{\varphi}^-) + \hat{h}^-)\cos(\hat{\varphi}^-)) \end{pmatrix}$$
(15)

To include the correlations between the actual and cloned states the state vector  $\Delta x$  of the kalman filter in equation 1 is extended with the two cloning states

$$\Delta \hat{x}_c = (\Delta north_c, \Delta east_c)^T.$$
(16)

To preserve the correlation between the cloned states, the system covariance matrix  $P_{xx}$  is extended for the covariance matrix of the cloned states  $P_{cc}$  and their cross-correlation covariance matrix  $P_{cx}$ ,  $P_{xc}$ . The new system covariance matrix at time index k is

$$\check{\mathbf{P}}_{k} = \begin{pmatrix} \mathbf{P}_{xx,k} \, \mathbf{P}_{xc,k} \\ \mathbf{P}_{cx,k} \, \mathbf{P}_{cc,k} \end{pmatrix} \tag{17}$$

and the extended system model with transition matrix  $\Phi$  and process noise matrix  ${\bf G}$  can be described as

$$\check{x}_{k} = \begin{pmatrix} x_{k+1} \\ x_{k+1}^{c} \end{pmatrix} = \underbrace{\begin{pmatrix} \mathbf{\Phi} \mathbf{0} \\ \mathbf{0} \mathbf{I} \\ \check{\mathbf{\Phi}}_{k} \end{pmatrix}}_{\check{\mathbf{\Phi}}_{k}} \cdot \begin{pmatrix} x_{k} \\ x_{k}^{c} \end{pmatrix} + \underbrace{\begin{pmatrix} \mathbf{G}_{k} \\ \mathbf{0} \\ \check{\mathbf{G}}_{k} \end{pmatrix}}_{\check{\mathbf{G}}_{k}} \cdot w_{k}$$
(18)

It is obvious that the cloned state is not propagated (stationary state). Only the original states are propagated as before (evolving states). A state cloning step is performed by

$$x_k^c = x_k \tag{19}$$

$$\mathbf{P}_{cc,k} = \mathbf{P}_{xx,k}(1:2,1:2)$$
(20)

$$\mathbf{P}_{xc,k} = \mathbf{P}_{xx,k}(1:17,1:2)$$
 (21)

$$\mathbf{P}_{cx,k} = \mathbf{P}_{xx,k}(1:2,1:17) \tag{22}$$

The state propagation is done by using the system model in 18. The standard kalman filter equations can be used with the system noise  $\mathbf{Q}$ .

$$\check{x}_{k+1}^- = \check{\Phi}_k \check{x}_k^+ \tag{23}$$

$$\check{\mathbf{P}}_{k+1}^{-} = \check{\mathbf{\Phi}}_{k}\check{\mathbf{P}}_{k}^{+}\check{\mathbf{\Phi}}_{k}^{T} + \check{\mathbf{G}}_{k}\check{\mathbf{Q}}_{k}^{+}\check{\mathbf{G}}_{k}^{T}$$
(24)

Propagation for state and covariance matrix can be calculated for every sub-matrix separately. The additional workload is minimal [1]. The measurement step of the stochastic cloning kalman filter refers to the actual and the cloned state. The standard kalman filter equations 2 can be used with the extended measurement matrix  $\tilde{\mathbf{H}}_k$ .

$$\dot{\mathbf{H}}_k = (\mathbf{H}_k, \mathbf{H}_k^c) \tag{25}$$

 $\mathbf{H}_k$  refers to the actual states and  $\mathbf{H}_k^c$  refers to the cloned states. If the cloned state is not part of the measurement model then  $\mathbf{H}_k^c$  is set to zero. For example in case of pseudo range measurements. Then the cloned states are corrected by the remaining cross correlations. For a step length update  $\check{\mathbf{H}}_k$  is

$$\check{\mathbf{H}}_{k} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & -1 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & -1 \end{pmatrix}$$
(26)

## IV. IMPLEMENTATION

In this section several implementation aspects of the Deeply Coupled GPS/PN System are described. First the software defined GNSS receiver platform is presented. Then three algorithmic parts,  $C/N_0$  and discriminator estimation, step length estimation and deeply acquisition are stated.

### A. software defined GNSS receiver

For a software defined GNSS receiver only an antenna and a HF-front-end including an A/D converter is necessary. All necessary signal processing is done in software. The software can run on different hardware like PCs, often the correlation process is supported by reconfigurable hardware (FPGA). In this work a HF-front end is used which streams raw data to a PC. The recorded data are processed with a Matlab software defined GNSS receiver. A real time processing is not possible but in contrast to some hardware implementations there are no internal timing delays to consider [3]. The HF frontend consist of a SiGe4120L chip set. The signal is complex sampled with  $f_A = 8.1838MHz$  and down converted to an intermediate frequency of  $f_{IF} = 38.500 kHz$ . The post processing is divided into several phases. At the acquisition phase a parallel code phase search is performed [4]. Tracking and positioning are done in parallel in contrast to [4]. During bit and frame decoding the signal is tracked by standard tracking loops. After ephemeris decoding and the calculation of an initial position the deeply mode is activated. This can be seen in Fig. 3.

The signal tracking and the generation of GPS observations is realized as in a hardware receiver. This means, the correlations are performed asynchronously over complete C/Acodes.  $1ms \ I, Q$  correlation values are available at the end of a complete code and  $20ms \ I, Q$  correlation values are available at the end of every  $20^{th}$  code (at the end of a bit).



Fig. 3. Sequencial flow of post processing. Green is deeply coupled mode



Fig. 4. Torso system with IMU, baro, magnetometer, laser ranger and camera

GPS observations are generated for all channels at the same time. This is in contrast to the implementation in [4]. Therefor the complexity is increased but the observations are generated in respect to the measurement model where an identical clock error for all measurements is assumed.

## B. pedestrian navigation system / step length estimation

For torso mounted pedestrian navigation systems, [5] proposes the use of a dead reckoning approach with a step length estimator. For heading information the use of a magnetic compass and a barometric height sensor for height estimation is proposed. For robustness in our approach, the heading angle is estimated not only with the compass readings but IMU measurements are used for leveling and integrity monitoring. We have developed a multi sensor pedestrian navigation system containing IMU, barometer and magnetic compass, and additional sensors like laser ranger and monocular camera. The system can be seen in Fig. 4.

In a dead reckoning system, first, steps are detected regarding down and forward acceleration. With each detected step from the torso IMU signal, a new step with a defined step length and heading is computed. The result is a polygon of 2D points (or 3D points if the barometric height sensor is in use). To define a step length, there are 2 possibilities: Either a fixed step length can be assumed or the step length is estimated based on the acceleration energy and step frequency of the current step [5]. The basic formula is

$$SL = \alpha \cdot f_{step} + \beta \cdot var(\vec{a}_{3D}) + \gamma \tag{27}$$

where SL is the estimated step length,  $f_{step}$  is the step frequency and  $var(\vec{a}_{3D})$  is the acceleration energy of a step. To estimate the parameters  $\alpha$ ,  $\beta$  and  $\gamma$ , a number of calibration walks, covering slow strolling as well as quicker walks with longer step lengths have to be performed. With a Least Squares Estimator, finally the step length parameters for one particular person can be computed. Of course the variable step length estimation needs to be calibrated for each user. For practical usage it is absolutely essential that this calibration is computed automatically with a given ground truth. We are able to calibrate our systems with both, a very accurate foot mounted system (indoor) or with GPS (outdoor).

Finally, the resulting polygon is used as a Step Length Update (SLU) in the state cloning kalman filter, which is described in chapter III.

#### C. $C/N_0$ and discriminator estimation

For signal tracking in weak signal conditions a reliably  $C/N_0$  estimation is necessary to weight the code and carrier discriminators. In [6] different  $C/N_0$  estimation methods and carrier and code discriminators have been examined especially regarding low signal levels. For a non-coherent tracking the early-late-power discriminator  $D_{ELP}$  was chosen. The code discriminator describes the code tracking error in equation 6.

$$D_{ELP} = (I_E^2 + Q_E^2) - (I_L^2 + Q_L^2).$$
(28)

 $I_E, Q_E$  and  $I_L, Q_L$  are the early and late I, Q correlation values. The discriminator has to be normalized. This can either be done by using the I, Q values or a  $C/N_0$  estimation [7].

$$D_{ELP,C/N_0} = \frac{(I_E^2 + Q_E^2) - (I_L^2 + Q_L^2)}{\sigma_T^2 \frac{2CT}{N_0} (4 - 2d)}$$
(29)

$$D_{ELP,IQ} = \frac{2-d}{4} \frac{(I_E^2 + Q_E^2) - (I_L^2 + Q_L^2)}{(I_E^2 + Q_E^2) + (I_L^2 + Q_L^2)}$$
(30)

The normalization factor includes the early-late spacing d and the integration time T. In case of low  $C/N_0$ , the values of the second discriminator are not representing the true code phase error any more [6] and the  $C/N_0$  normalized discriminator should be used. Using the  $C/N_0$  normalized discriminator the variance of the discriminator which is needed to weight the GPS measurements can be calculated with [8]

$$\sigma_{D_{ELP,C/N_0}}^2 = \frac{d}{4C/N_0T} \left(1 + \frac{2}{(2-d)C/N_0T}\right).$$
 (31)

The frequency error in 10 is described by a cross-product discriminator  $D_{CP}$ . In a cross-product discriminator (I, Q) values from time index k and a previous time index k - 1 have to be used. This limits the integration time T = 10ms if bit transitions are ignored [7].

$$D_{CP} = I_{P,k-1}Q_{P,k} - I_{P,k}Q_{P,k-1}$$
(32)

Again a  $C/N_0$  or an (I, Q) normalized discriminator can be stated [7].

$$D_{CP,C/N_0} = \frac{I_{P,k-1}Q_{P,k} - I_{P,k}Q_{P,k-1}}{\sigma_T^2 \frac{4CT^2}{N_0}\pi}$$
(33)

$$D_{atan2} = \frac{1}{2\pi T} atan2(I_{P,k-1}Q_{P,k} - I_{P,k}Q_{P,k-1}, I_{P,k-1}I_{P,k} - Q_{P,k}Q_{P,k-1})$$
(34)

The resulting variance for the  $C/N_0$  normalized discriminator is [6]

$$\sigma_{D_{CP,C/N_0}}^2 = \frac{1}{(2\pi)^2 T^3 C/N_0} \left(1 + \frac{1}{2TC/N_0}\right).$$
 (35)

The  $C/N_0$  estimation needed for normalizing the discriminators is realized by using the Narrow-to-wide-band power method (NWBPM) [9]. This method uses a power ratio  $P_{N/W}$ between a narrow-band  $P_N$  and a wide-band power  $P_W$ . They are calculated by using (I, Q) correlation values and can be written as

$$P_{N} = \left(\sum_{i=1}^{M} I_{P,i}\right)^{2} + \left(\sum_{i=1}^{M} Q_{P,i}\right)^{2}$$
(36)

$$P_W = \sum_{i=1}^{N} \left( I_{P,i}^2 + Q_{P,i}^2 \right)$$
(37)

$$P_{N/W} = P_N/P_W \tag{38}$$

In this case M equals 20 which results in an integration interval of one bit length or  $T_M = 20ms$  respectively. The power ratio can be averaged over several measurements

$$\overline{P}_{N/W} = \frac{1}{n} \sum_{i=1}^{n} (P_{N/W})_i.$$
(39)

The  $C/N_0$  can then be estimated with

$$\frac{\hat{C}}{\hat{N}_0} \approx \frac{M}{T_M} \frac{\overline{P}_{N/W} - 1}{M - \overline{P}_{N/W}}.$$
(40)

Test results show that a reliable  $C/N_0$  estimation is necessary for a robust Deeply Coupled System especially in weak signal conditions. During phases of low signal receptions (I, Q)normalized discriminators are not unbiased [6] and lead to false weightings of GPS measurements. Estimation results can be further improved by using an adaptive kalman filter for  $C/N_0$  estimation. In urban environments signal energy can change abrupt and significantly. A kalman filter can be used to detect these jumps and therefore improve the estimation of the  $C/N_0$ .

#### D. deeply acquisition

In urban environments a GPS receiver has to cope with changing signal levels and for example completely shadowed satellites. A Deeply Couples GPS/INS System can not only bridge outages of satellites but can also be used at acquisition stage. Before the deeply mode can be used an initial acquisition has to be performed. Depending on acquisition method, integration length, et cetera only a subset of available satellites



Fig. 5. Sky plot of scenario presented in section V. GDOP and number of satellites before and after deeply acquisition

may be found. If ephemeris data of additional satellites and a initial position solution is available these satellites can be acquired instantly. Normally the satellite position can be calculated using the measured transmission time or pseudo range. For a instant acquisition the time of signal transmission and the satellite position at the time of transmission have to be calculated. This is done using an iterative process since the two values depend on each other. By performing the following steps the time of signal transmission can be calculated.

- 1) Initial position at time of signal arrival  $t_{sa}$
- 2) Initial signal propagation time  $\tau_0$  estimation (For example 75ms)
- 3) Calculation of time of signal transmission  $t_{st} = t_{sa} \tau_{n-1}$
- Calculation of satellite position at time of signal transmission in ECEF coordinates.
- 5) Calculation of satellite position at time of signal transmission in ECEF coordinates at time of signal arrival using  $\tau_{n-1}$ .
- 6) Calculation of  $\tau_n$  using estimated pseudo range after clock error correction.
- 7) Repeat 3. 6. until  $c_0 \cdot |\tau_{n-1} \tau_n| < \epsilon$ .

After a view iterations a precision of  $\epsilon = 10^{-6}$  is reached. The satellite velocity can be calculated without further iterations. Step 5. is needed since the earth is rotating relative to the ECEF frame during signal propagation. With satellite velocity and receiver clock error drift estimation the doppler frequency of the new satellite for the tracking loop of the GPS receiver can be set. The code phase can be extracted from the estimated time of signal transmission.

Fig. 5 shows the deeply acquisition for the scenario presented in section V. Instead of seven satellites, ten satellites can be used instantly after the first position fix (new satellites: 2, 29, 31). As a result the GDOP is improved. Also the  $C/N_0$ of these satellites is lower the position solution is improved especially during periods where the signal energy is rising (see Fig. 8).

### V. RESULTS

In this section the results of the Deeply Coupled GPS/PN System are shown. The trajectory was recorded at the campus of the Karlsruhe Institute of Technology (KIT) using the USB module described in section IV-A and the torso module of the pedestrian navigation system. Actual ephemeris data have



Fig. 6. Test run front-end SiGe4120L, red: Deeply Coupled GPS/PN, green: VDFLL, blue: SLU DR, cyan: reference trajectory

been recorded at a rooftop of a KIT building previously to allow a deep acquisition of satellites with low  $C/N_0$ .

The results can be seen in Fig. 6. The run starts from east to west. At the beginning a transient response of the kalman filter can be observed. After a first fix the deeply mode is activated and additional satellites are acquired. The sky plot in Fig. 5 corresponds to this scenario. The way leads between two high buildings and crosses a building by walking a right and left turn. Leaving the building on the other side, the path leads north after a second right turn. The approximate reference trajectory (cyan) was added afterwards because of the absence of a ground truth. The performance of the vector receiver (green) using a DLL-FLL approach performs similar compared to the Deeply Coupled GPS/PN approach (red) when walking outdoors. Inside the building the results are more noisy nevertheless a position solution is still possible. After leaving the building the transient response is stronger. The trajectory of the Deeply Coupled GPS/PN System is much smother and especially inside the building a much more accurate position solution is available. Due to the aiding through SLU there is a clean transient when the weight of the GPS measurements is raising again after leaving the building. For comparison the dead reckoning position solution after a first fix is displayed in blue. A constant north drift can be observed.

In Fig. 7 the transient region between outdoor - indoor outdoor and the advantage of the Deeply Coupled GPS/PN approach can be seen in detail. Using a vector receiver a position solution inside the building is still available but does not follow the reference trajectory well. Especially the two turns are not clearly visible. The Deeply Coupled GPS/PN System is able to follow the reference trajectory using the SLU information. By being able to track the satellite signals inside the building and a reasonable weighting of the GPS measurements the drift of the dead reckoning can be corrected.



Fig. 7. Test run front-end SiGe4120L (zoomed in), red: Deeply Coupled GPS/PN, green: VDFLL, blue: SLU DR, cyan: reference trajectory



Fig. 8.  $C/N_0$  estimation of visible satellites of the Deeply Coupled GPS/PN System

The  $C/N_0$  estimation is plotted in Fig. 8. In can be seen that satellite 2, 29, 31 are acquired after a first fix. Between 50s and 120s when walking through the shadow of the two buildings the  $C/N_0$  drops to  $\approx 25 - 30dBHz$ . Inside the building the  $C/N_0$  is below 25dBHz, a standard stand alone GPS receiver would loose track. After leaving the building all satellites are still in track and the  $C/N_0$  for all signals raises immediately.

## VI. CONCLUSION

In this paper we have presented a Deeply Coupled INS/GPS system, extended by step length updates. Furthermore we have identified different aspects which are crucial to implement a robust and reliably Deeply Coupled GPS/PN System which can work on real world data. Especially the transition between indoor and outdoor has to be handled carefully due to the direct feedback of the position solution into the tracking algorithms. Additionally using a deeply acquisition of satellites with a very low  $C/N_0$  is possible. This is important especially in urban environments where signal strength can fluctuate rapidly. The including of step length updates improved the position solution significantly. Using this technique it is possible to reduce the drift of the inertial navigation indoors and to improve the position solution outdoors. Due to the inertial aiding of the tracking loops GPS measurements are available instantly when a satellite comes in few again. This also improves the overall position solution.

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